A Timer-Based Distributed Channel Access Mechanism in Networked Control Systems

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The NCS consists of $N$ dynamical subsystems sharing the limited communication resources.

Each subsystem contains **smart sensors** which are capable of local computations.

Local measurements are sent through the network to be received by their corresponding estimator.
System and Network Models

- Each subsystem is modeled by a discrete-time LTI stochastic process

\[ x_{i,k+1} = A_i x_{i,k} + B_i u_{i,k} + w_{i,k}, \]
\[ y_{i,k} = C_i x_{i,k} + v_{i,k}, \]

- Variable \( \delta_{i,k} \) denotes whether the measurements of subsystem \( i \) is transmitted or not

\[ \delta_{i,k} = \begin{cases} 1, & y_{i,k} \text{ is transmitted,} \\ 0, & \text{otherwise.} \end{cases} \]

- The communication channels are assumed to be reliable, therefore if no collision happens

\[ \delta_{i,k} = 1 \rightarrow y_{i,k} \text{ is received at destination} \]
Estimator

- Define

\[
\begin{align*}
\hat{x}_{i,k+1|k} &= \mathbb{E}\{x_{i,k+1|Y_k}\}, \\
P_{i,k+1|k} &= \mathbb{E}\{(x_{i,k} - \hat{x}_{i,k+1|k})(x_{i,k} - \hat{x}_{i,k+1|k})^T|Y_k\}, \\
\hat{x}_{i,k+1|k+1} &= \mathbb{E}\{x_{i,k+1|Y_{k+1}}\}, \\
P_{i,k+1|k+1} &= \mathbb{E}\{(x_{i,k+1} - \hat{x}_{i,k+1|k+1})(x_{i,k+1} - \hat{x}_{i,k+1|k+1})^T|Y_{k+1}\}.
\end{align*}
\]

- Kalman filter [Sinopoli et al., 2004]

\[
\begin{align*}
\hat{x}_{i,k+1|k} &= A_i\hat{x}_{i,k|k} + B_iu_{i,k}, \\
P_{i,k+1|k} &= A_iP_{i,k|k}A_i^T + W_i, \\
K_{i,k+1} &= P_{i,k+1|k}C_i^T \left(C_iP_{i,k+1|k}C_i^T + V_i\right)^{-1}, \\
\hat{x}_{i,k+1|k+1} &= \hat{x}_{i,k+1|k} + \delta_{i,k+1}K_{i,k+1}(y_{i,k+1} - C_i\hat{x}_{i,k+1|k}), \\
P_{i,k+1|k+1} &= (I - \delta_{i,k+1}K_{i,k+1}C_i)P_{i,k+1|k}.
\end{align*}
\]
Introduction

Controller

- The cost function is defined as

\[ J_{i,0} = \mathbb{E} \left\{ \sum_{k=0}^{\infty} (x_{i,k}^T Q_i x_{i,k} + u_{i,k}^T R_i u_{i,k}) \right\} \]

- The control law which minimizes this cost is

\[ u_{i,k} = L_i \hat{x}_{i,k} |_{k} \]

where

\[ L_i = - (B_i^T \Pi_i B_i + R_i)^{-1} B_i^T \Pi_i A_i \]

\[ \Pi_i = A_i^T \Pi_i A_i - A_i^T \Pi_i B_i (B_i^T \Pi_i B_i + R_i)^{-1} B_i^T \Pi_i A_i + Q_i \]

- The feedback gain \( L_i \) does not depend on scheduling or characteristics of the disturbances
Every subsystem possesses a local timer which is set to

\[ \tau_{i,k} = \frac{\lambda}{m_{i,k}} \]

The timers start at the beginning of each time slot and the subsystem whose timer finishes first begins transmission after sending a flag packet.
Distributed Channel Access Mechanism

Concept of Local Timers

- Every subsystem possesses a local timer which is set to

\[ \tau_{i,k} = \frac{\lambda}{m_{i,k}} \]

- The timers start at the beginning of each time slot and the subsystem whose timer finishes first begins transmission after sending a flag packet.
Distributed Channel Access Mechanism

Cost of Information Loss (CoIL)

• Minimizing the quadratic cost at each step $k$ is equivalent to

$$\text{minimize } \sum_{i=1}^{N} tr \left( \Gamma_{i,k} P_{i,k|k} \right),$$

where $\Gamma_{i} = L_{i}^{T} (B_{i}^{T} \Pi_{i} B_{i} + R_{i}) L_{i}$.

• According to Kalman filter equations

$$P_{i,k|k} = \begin{cases} (I - K_{i,k} C_{i}) P_{i,k|k-1}, & \delta_{k} = 1 \\ P_{i,k|k-1}, & \delta_{k} = 0 \end{cases}$$

Hence, the increase in total cost in case subsystem $i$ does not transmit at step $k$ is defined as CoIL [Charalambous et al., 2017]

$$\text{CoIL}_{i,k} = tr \left( \Gamma_{i} (P_{i,k|k-1} - P_{i,k|k}) \right),$$
Algorithm 1: TBCoIL for general case of $c$ available channels

Input: $c, \lambda$

1 Initialization: Start timers from $\tau_j = \frac{\lambda}{\text{CoIL}}$ and let number of flags $f = 0$

2 while $f < c$ do

3 for $j \leftarrow 1$ to $c$ do

4 if $\tau_j \neq 0$ and timer is running then

5 listen for flags

6 if flag is received then

7 freeze $\tau_j$ and $f = f + 1$

8 end

9 else if $\tau_j = 0$ then

10 send flag and set $f = c$

11 freeze all running timers

12 end

13 end

14 end

15 if any $\tau_j = 0$ then transmit on channel $j$
Numerical Results

Illustrative Example

- 3 subsystems have access to 1 channel
- subsystem 1 is stable while 2 & 3 are unstable
- $\lambda = 52.82 \times 10^{-3} m^2 s$
Numerical Results

Numerical Results 1

- The subsystems are selected from two homogeneous classes of unstable (I) and stable (II)

\[
A_I = \begin{bmatrix} 1.1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_{II} = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad B = C = I_{2 \times 2}.
\]

- Only 1/2 subsystems can transmit their measurements at each step
- Round-robin protocol is used as the basis of comparison
The subsystems are selected from two homogeneous classes of stable (I) and unstable (II)

\[ A_I = \begin{bmatrix} 1.4 & 0 \\ 0 & 1.2 \end{bmatrix}, \quad A_{II} = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad B = C = I_{2 \times 2}. \]

Only 1/2 subsystems can transmit their measurements at each step.

Round-robin protocol is used as the basis of comparison.
Summary

• Novel distributed channel access mechanism was introduced
  ◦ Concept of local timers for general case of prioritization

• CoIL was used as the local measure for implementation

• Distributed implementation was shown to perform as well as the optimal central scheduler

Future directions

• Possibility of collision due to non-negligible duration of flag packets

• Extension to the case of wireless communication and presence of imperfect channels
Conclusion and future work

Thank You!

Questions?