Transmission Scheduling in Wireless Networks with SINR Constraints

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April 22, 2009
Outline

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- Motivation
- Medium Access Control (MAC)

2 System Model
- Notation
- The network
- The channel

3 Main Results
- Problem Definition
- Assumptions
- Formulation - Solution
- Examples

4 Conclusions
- Conclusions
- Future Directions
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**Applications:**
- Battlefield communications
- Disaster recovery efforts
- Impromptu communication between people
- Wireless traffic sensor networks
- Ecological habitat monitoring
- Industrial process control

**Why transmission scheduling?**
Orchestrate channel access in order to fully exploit spatial reuse:
- Establish feasible networks for successful transmissions
- Minimize the number of time-slots required
- Minimize the power dissipated in the network
Medium Access Control (MAC) - 1/2

MAC: Part of the Data Link Layer (layer 2 of the OSI model)

- Sits directly on top of the Physical Layer (layer 1)
- Purpose: to manage access to the shared wireless medium
Nodes must decide when to access the channel, *avoiding collisions* and *efficiently utilizing the bandwidth*.

**Classification of MAC protocols:**
- **Random access**
  - Nodes contend for the channel
  - Most popular is CSMA/CA, the basis for IEEE 802.11
- **Transmission Scheduling**
  - Time is divided into frames and frames into time-slots.
  - Simplest example is TDMA (as many slots as nodes, one node per slot).
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<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>The set of all nodes in the network</td>
</tr>
<tr>
<td>$\mathcal{L}$</td>
<td>The set of active links in the network</td>
</tr>
<tr>
<td>$G = (N, \mathcal{L})$</td>
<td>The graph of the network</td>
</tr>
<tr>
<td>$T$</td>
<td>The set of transmitters in the network</td>
</tr>
<tr>
<td>$R$</td>
<td>The set of receivers in the network</td>
</tr>
<tr>
<td>$g_{ij}$</td>
<td>The channel gain on the link between transmitter $i$ and receiver $j$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>The variance of thermal noise at the receiver</td>
</tr>
<tr>
<td>$I_i$</td>
<td>The interference power at the $i^{th}$ receiver</td>
</tr>
<tr>
<td>$\Gamma_i$</td>
<td>The SINR at the $i^{th}$ receiver</td>
</tr>
<tr>
<td>$\gamma_i$</td>
<td>The capture ratio set by the $i^{th}$ receiver</td>
</tr>
<tr>
<td>$P_{i}^{\text{max}}$</td>
<td>The maximum power capacity of the $i^{th}$ transmitter</td>
</tr>
<tr>
<td>$D$</td>
<td>The deadline of the network</td>
</tr>
</tbody>
</table>
The network

Planar network

Half-duplex transceivers ⇒ Unidirectional links

Omnidirectional antennas

Figure: An example of an ad hoc topology.
The simultaneous operation of wireless links usually is characterized by models that assume fixed interference and communication ranges, such as

1. The *Ideal Model*
2. The *Protocol Model*

On the contrary, the *Physical Model* is more realistic, since it considers the cumulative effects of interference due to simultaneous transmissions. This model considers the Signal-to-Interference-and-Noise Ratio (SINR) constraints at the receivers by considering all the secondary transmissions as noise.
The simultaneous operation of wireless links usually is characterized by models that assume fixed interference and communication ranges, such as:

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We consider the *Physical Model* where receivers experience interference:

\[
I_i = \sum_{j \neq i, j \in \mathcal{T}} g_{ji} p_j + \nu. \tag{1}
\]

where

- \(g_{ij}\): the channel gain on the link between transmitter \(i\) and receiver \(j\).
- \(p_i\): the power level chosen by transmitter \(i\).
- \(\nu\): the variance of thermal noise at the receiver.

The link quality is measured by the *Signal-to-Interference-and-Noise-Ratio* (SINR). A transmission is successful (error free), if the SINR at the receiver \((\Gamma_i)\) is greater than the *capture ratio*, \(\gamma_i\). Therefore, we require,

\[
\Gamma_i = \frac{g_{ii} p_i}{\sum_{j \neq i, j \in \mathcal{T}} g_{ji} p_j + \nu} \geq \gamma_i \tag{2}
\]
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Problem Definition

Problem

Given $N$ communication requests, assign a color (time-slot) to each request. For all requests sharing the same color specify the power levels such that each request can be handled correctly.
Assumptions

- No node mobility or routing is considered.
- Time is slotted and each communication pair needs exactly one slot.
- Nodes maintain global synchronization: They know when slots and frames start.
Problem formulation

Minimize

\[ \tau = \max_{i \in \mathcal{I}} \left\{ \sum_{t=0}^{D} tx_i(t) \right\} \]  

subject to:

\[ \sum_{t=0}^{D} x_i(t) \geq 1 \] \hspace{1cm} \forall i \in \mathcal{I} \quad (4)

\[ p_i(t) \leq x_i(t)P_i^{\text{max}} \] \hspace{1cm} \forall i \in \mathcal{I}, t \in [0, D] \quad (5)

\[ p_i(t)g_{ii} \geq \gamma_i \left\{ \sum_{j \neq i, j \in \mathcal{I}} g_{ji}p_j(t) + \nu \right\} - (1 - x_i(t))M_i \] \hspace{1cm} \forall i \in \mathcal{I}, t \in [0, D] \quad (6)

\[ x_i(t) \in \{0, 1\} \] \hspace{1cm} \forall i \in \mathcal{I}, t \in [0, D] \quad (7)

\[ p_i(t) \geq 0 \] \hspace{1cm} \forall i \in \mathcal{I}, t \in [0, D] \quad (8)
Solution

- Coded in Microsoft Visual Studio 2005 $C++$ using CPLEX v11.1
- Intel Core 2 computer (RAM: 3.5GB, Processor: 2.5GHz)

**ICNS paper:**

1. Solution approach: a non problem specific Branch and Bound (B&B) algorithm.

**Improvements:**

1. Solution approach: Exact cutting plane algorithm with lower and upper bounding techniques.
2. Optimal schedule such that total power is minimized.
### Illustrative example - 1/2

<table>
<thead>
<tr>
<th>Time Slot</th>
<th>Links in process</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5 (15820.5), 8 (385019), 9 (579226)</td>
</tr>
<tr>
<td>2</td>
<td>3 (17739.8), 4 (763932), 7 (355380)</td>
</tr>
<tr>
<td>3</td>
<td>2 (206559), 6 (24638.4)</td>
</tr>
<tr>
<td>4</td>
<td>1 (2924.78), 10 (166.464)</td>
</tr>
</tbody>
</table>
Problem Definition

Assumptions

Formulation - Solution

Examples

Illustrative example - 2/2

The validity of the power levels can be found in a centralized way by

\[
p^* = (I - C_s)^{-1} \eta_{(27)}
\]

where \( s \) is the time-slot considered; \( p^* \) and \( C_s \) are the vector of optimal power levels and the matrix formed by the subnetwork on time-slot \( s \), respectively. However, once the scheduling problem is solved and the information required is sent to the communication pairs, the Foschini-Miljanic Power Control algorithm, [11], succeeds in attaining the required SINRs for all pairs in the subnetwork in a distributed manner. We demonstrate this, by simultaneously verifying the validity of our results, in Figure 3.

For our computational study we generated a set of test networks of various sizes which will serve as the benchmark for evaluating our B&B algorithm. More specifically, the benchmark set consists of networks with 10, 20, 30 and 40 pairs (we have 5 instances of each) and hence, in total, we have tested the algorithm on 20 networks. Table II shows the average computational time (in CPU seconds) required to solve the network instances as well as the percentage of instances which were solved optimally within the time limit set. We chose 3600 CPU seconds as a time limit. Note that the default settings of CPLEX were used to solve the optimization problem and no problem specific strategies to drive the solver were used.

<table>
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<th>No. of pairs</th>
<th>CPU time (sec)</th>
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<tr>
<td>10</td>
<td>0.250</td>
</tr>
<tr>
<td>20</td>
<td>8.797</td>
</tr>
<tr>
<td>30</td>
<td>73.126</td>
</tr>
<tr>
<td>40</td>
<td>75.409</td>
</tr>
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In Table II the results of our algorithm are shown. As expected, the larger the number of communication pairs considered in the network, the more difficult to solve the problems. While networks of 10 pairs are optimally solved in less than one second, networks of 20 pairs require 376 seconds on average, 30 pairs require 1318 seconds (with one not being
Numerical examples

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<tr>
<td></td>
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</tr>
<tr>
<td>10</td>
<td>0.250 0.407</td>
</tr>
<tr>
<td>20</td>
<td>8.797 36.829</td>
</tr>
<tr>
<td>30</td>
<td>73.126 178.204</td>
</tr>
<tr>
<td>40</td>
<td>75.409 900.033</td>
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Exponential growth of complexity with the number of communication pairs.
Numerical examples

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Conclusions

- Solved the general transmission scheduling problem
  - Under SINR constraints.
  - Transmitters are able to adjust their power levels.

- Significance is twofold:
  - Of practical importance when a centralized controller exists.
  - Provides bounds for the evaluation of distributed algorithms.
Future Work

- **Improve centralized algorithm:**
  - Find a fast and efficient methodology. ✓
  - Make it problem specific. ✓
  - Find the scheduling that minimizes the total power as well. ✓

- **Consider the geometrical physical model where:**
  - Attenuation coefficients depend only on distance.
  - Information on the topology can be extracted.

- **Distributed algorithm:**
  - So far, a central entity makes decisions, based on global topology information. Not very useful for large networks since *complexity* and *overhead communication* increases.
  - *Scalability*: extremely important, as wireless ad hoc networks are expected to be potentially large.