A Timer-Based Distributed Channel Access Mechanism in Networked Control Systems

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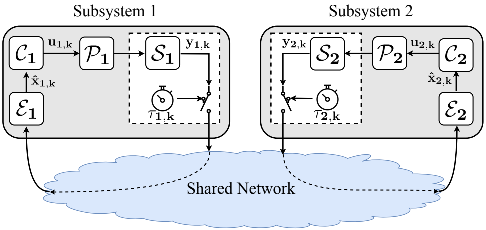
Networked Control Systems

- The NCS consists of N dynamical subsystems sharing the limited communication resources
- Each subsystem contains **smart sensors** which are capable of local computations
- Local measurements are sent through the network to be received by their corresponding estimator



System and Network Models

- Each subsystem is modeled by a discrete-time LTI stochastic process

$$\begin{aligned} x_{i,k+1} &= A_i x_{i,k} + B_i u_{i,k} + w_{i,k}, \\ y_{i,k} &= C_i x_{i,k} + v_{i,k}, \end{aligned}$$

- Variable $\delta_{i,k}$ denotes whether the measurements of subsystem *i* is transmitted or not

$$\delta_{i,k} = \begin{cases} 1, & y_{i,k} \text{ is transmitted,} \\ 0, & \text{otherwise.} \end{cases}$$

- The communication channels are assumed to be reliable, therefore if no collision happens

$$\delta_{i,k} = 1 \rightarrow y_{i,k} \text{ is received at desitnation}$$

Estimator

- Define

$$\begin{aligned}
\hat{x}_{i,k+1|k} &= \mathbb{E}\{x_{i,k+1}|\mathcal{Y}_k\},\\
P_{i,k+1|k} &= \mathbb{E}\{(x_{i,k} - \hat{x}_{i,k+1|k})(x_{i,k} - \hat{x}_{i,k+1|k})^T|\mathcal{Y}_k\},\\
\hat{x}_{i,k+1|k+1} &= \mathbb{E}\{x_{i,k+1}|\mathcal{Y}_{k+1}\},\\
P_{i,k+1|k+1} &= \mathbb{E}\{(x_{i,k+1} - \hat{x}_{i,k+1|k+1})(x_{i,k+1} - \hat{x}_{i,k+1|k+1})^T|\mathcal{Y}_{k+1}\}.
\end{aligned}$$

- Kalman filter [Sinopoli *et al.*, 2004]

$$\begin{aligned}
\hat{x}_{i,k+1|k} &= A_i\hat{x}_{i,k|k} + B_i u_{i,k},\\
P_{i,k+1|k} &= A_i P_{i,k|k} A_i^T + W_i,\\
K_{i,k+1} &= P_{i,k+1|k} C_i^T \left(C_i P_{i,k+1|k} C_i^T + V_i\right)^{-1},\\
\hat{x}_{i,k+1|k+1} &= \hat{x}_{i,k+1|k} + \delta_{i,k+1} K_{i,k+1}(y_{i,k+1} - C_i\hat{x}_{i,k+1|k}),\\
P_{i,k+1|k+1} &= (I - \delta_{i,k+1} K_{i,k+1} C_i) P_{i,k+1|k}.
\end{aligned}$$

Controller

- The cost function is defined as

$$J_{i,0} = \mathbb{E}\left\{\sum_{k=0}^{\infty}\left(x_{i,k}^T Q_i x_{i,k} + u_{i,k}^T R_i u_{i,k}\right)\right\}$$

- The control law which minimizes this cost is

$$u_{i,k} = L_i \hat{x}_{i,k|k}$$

where

$$L_i = -(B_i^T \Pi_i B_i + R_i)^{-1} B_i^T \Pi_i A_i$$
$$\Pi_i = A_i^T \Pi_i A_i - A_i^T \Pi_i B_i (B_i^T \Pi_i B_i + R_i)^{-1} B_i^T \Pi_i A_i + Q_i$$

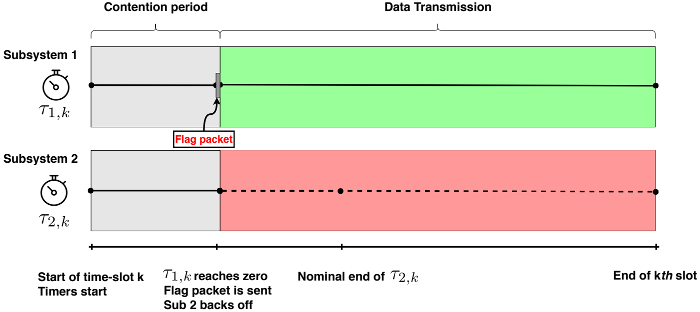
- The feedback gain L_i does not depend on scheduling or characteristics of the disturbances

Concept of Local Timers

- Every subsystem possesses a local timer which is set to

$$\tau_{i,k} = \frac{\lambda}{m_{i,k}}$$

- The timers start at the beginning of each time slot and the subsystem whose timer finishes first begins transmission after sending a flag packet



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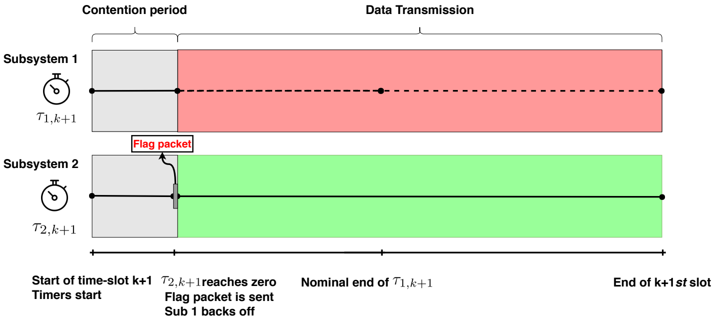


Cost of Information Loss (CoIL)

- Minimizing the quadratic cost at each step k is equivalent to

$$\text{minimize} \quad \sum_{i=1}^{N} tr\left(\Gamma_{i,k} P_{i,k|k}\right),$$

where $\Gamma_i = L_i^T (B_i^T \Pi_i B_i + R_i) L_i$.

- According to Kalman filter equations

$$P_{i,k|k} = \begin{cases} (I - K_{i,k} C_i) P_{i,k|k-1}, & \delta_k = 1 \\ P_{i,k|k-1}, & \delta_k = 0 \end{cases}$$

Hence, the increase in total cost in case subsystem i does not transmit at step k is defined as CoIL [Charalambous *et al.*, 2017]

$$\mathrm{CoIL}_{i,k} = \mathrm{tr}\left(\Gamma_i (P_{i,k|k-1} - P_{i,k|k})\right),$$

Timer-based Mechanism (TBCoIL)

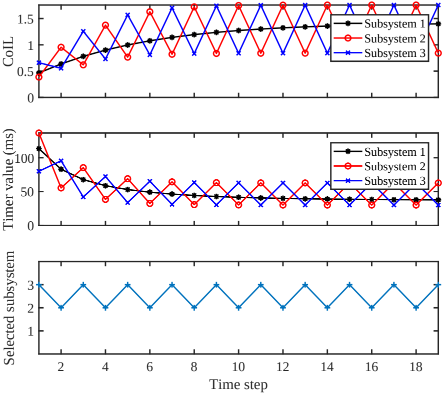
Algorithm 1: TBCoIL for general case of c available channels

Input: c, λ

Initialization: Start timers from $\tau_j = \frac{\lambda}{\mathrm{CoIL}}$ and let number of flags $f = 0$
while $f < c$ **do**
  for $j \leftarrow 1$ **to** c **do**
    if $\tau_j \neq 0$ *and timer is running* **then**
      listen for flags
      if *flag is received* **then**
        freeze τ_j and $f = f + 1$
      end
    else if $\tau_j = 0$ **then**
      send flag and set $f = c$
      freeze all running timers
    end
  end
end
if *any* $\tau_j = 0$ **then** transmit on channel j

Illustrative Example

- 3 subsystems have access to 1 channel
- subsystem 1 is stable while 2 & 3 are unstable
- $\lambda = 52.82 \times 10^{-3} m^2 s$

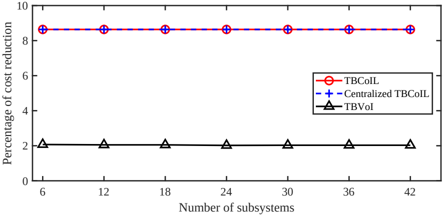


Numerical Results 1

- The subsystems are selected from two homogeneous classes of unstable (I) and stable (II)

$$A_I = \begin{bmatrix} 1.1 & 0 \\ 0 & 1 \end{bmatrix}, A_{II} = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}, B = C = I_{2\times 2}.$$

- Only $1/2$ subsystems can transmit their measurements at each step
- Round-robin protocol is used as the basis of comparison



Numerical Results 2

- The subsystems are selected from two homogeneous classes of stable (I) and unstable (II)

$$A_I = \begin{bmatrix} 1.4 & 0 \\ 0 & 1.2 \end{bmatrix}, A_{II} = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}, B = C = I_{2\times 2} .$$

- Only $1/2$ subsystems can transmit their measurements at each step
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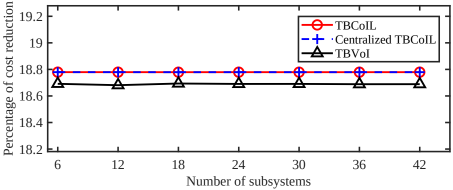

Summary

- Novel distributed channel access mechanism was introduced
  - Concept of local timers for general case of prioritization
- CoIL was used as the local measure for implementation
- Distributed implementation was shown to perform as well as the optimal central scheduler

Future directions

- Possibility of collision due to non-negligible duration of flag packets
- Extension to the case of wireless communication and presence of imperfect channels

Thank You!

Questions?