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RESEARCH ARTICLE

On the Minimum Latency Transmission Scheduling in Wireless Networks with Power Control under SINR Constraints†*

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ABSTRACT

In order to alleviate interference and contention in a wireless network, we may exploit the existence of multiple orthogonal channels or time slots, thus achieving a substantial improvement in performance. In this paper we study a joint transmission scheduling and power control problem that arises in wireless networks. The goal is to assign channels (or time slots) and transmitting powers to communication links such that all communication requests are processed correctly, specified Quality-of-Service (QoS) requirements are met, and the number of required time slots is minimised. First, we formulate the problem as a mixed-integer linear programming (MILP). Then, we show that the problem considered is NP-hard and subsequently, we propose non-trivial bounding techniques to solve it. Optimisation methods are also discussed, including a column generation approach, specifically designed to find bounds for the transmission scheduling problem. Moreover, we develop optimisation techniques in which the bounding techniques are integrated in order to derive the optimal solution to the problem faster. We close with an extensive computational study, which shows that despite the complexity of the problem, the proposed methodology scales to problems of nontrivial size. Our algorithms can therefore be used for static wireless networks where propagation conditions are almost constant and a centralised agent is available (e.g. cellular networks where the base station can act as a centralised agent or wireless mesh networks), and they can also serve as a benchmark for the performance evaluation of heuristic, approximation or distributed algorithms that aim to find near-optimal solutions without information about the whole network. Copyright © 2012 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Wireless technology standards provide a radio-frequency (RF) spectrum with a set of many non-overlapping channels, and a node has the option to choose on which channel to transmit. Likewise, in cases where only a single channel is available, it is possible to divide time into frames, and then frames can be divided into time slots, such that at each frame a node has the option to choose on which time slot to transmit. In the latter case, synchronisation of the wireless nodes in the network is necessary. If synchronisation is not considered, however, choosing a channel or a time slot in the network becomes the same problem. It is important to schedule channel/time slot access in such a way so that spatial reuse is fully exploited and hence the number of channels required to successfully complete all requests is minimised.

Power control has been a prominent research area with increased interest (e.g. [2–9]). Increased power ensures longer transmission distance and higher data transfer rate. However, power minimisation not only increases battery lifetime, but also the effective interference mitigation that increases the overall network capacity by allowing higher frequency reuse. Power control has been extensively employed for MAC in multi-hop wireless networks (for example in [10–15]). Some of them aim to minimise power dissipation. For example, [16] proposes a two-phase method for the joint scheduling and power control which aims to find an admissible set of links along with their transmission power levels in a single channel

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only. [17] study the same problem as in [16], but it focuses on minimising the scheduling length. Others, such as [18] and [11], aim to maximize throughput at the cost of increased power dissipation by allowing many simultaneous interference-limited transmissions. In such schemes either time is divided into fixed-length slots or there exist many channels and the wireless nodes have to choose on which one to transmit. The minimum latency scheduling problem (see for example [19–21] and reference therein) and the computation of efficient schedules for the abstract physical model with power control (for example in [10, 11, 14–16, 22–24]) have been both extensively studied. However, many approaches concentrate, for example, on throughput maximisation in a single or multiple channels (e.g., [25, 26]) and scheduling length minimisation (e.g., [27, 28] and reference therein), rather than the minimisation of the number of channels or time-slots required. A wide range of applications for wireless networks are time-critical and impose stringent requirement on the communication latency. For example, a given rate demand is requested (that should be satisfied with minimum-length periodic scheduling actions) or a given volume of traffic must be delivered to the destinations in minimum time. However, minimising the scheduling length requires coordination between the wireless nodes in order to orchestrate the order, duration and initialisation of transmissions, something which introduces extra communication overheads. On the other hand, minimum latency transmission scheduling does not require the communication between the nodes, but simply synchronisation to a global clock in the network, so that the nodes are able to know the beginning and the end of slots.

When studying wireless networks, the choice of model is crucial. Not only must the chosen model facilitate the design of protocols, but it also has to truthfully reflect the nature of the real network. Fading-channel models depict real-world phenomena in wireless communications. These phenomena include multi-path fading, shadowing and attenuation with distance. While fading effects have been considered as detrimental in 2G wireless networks, in 3G networks they are seen as an opportunity to increase the capacity that incorporate data traffic [29]. The most common fading-channel model being used is the physical model, which is thoroughly described in Section 2. On a finer granularity, one distinguishes between the *geometric* and the *abstract* physical model. In the geometric physical model the channel gain between two nodes is solely determined by their spatial distance. Hence, simplifying assumptions are incorporated into this model, for example, the radios are perfectly isotropic and there are no obstructions [30]. In the abstract physical model the channel gain between two nodes incorporates all the real-world phenomena and hence no information can be extracted about the geometry of the network.

The rest of the section concentrates on the related work that considers transmission scheduling under the (geometric and abstract) physical model only. For the geometric physical model, the NP-hardness of wireless scheduling without power control is proven in [30]. In [31] it is proven that strategies in the geometric physical model that use uniform power assignment schemes (same power to all nodes in the network) or linear power assignment schemes (power levels proportional to the minimum power required to reach the receiver node) have a bad scheduling complexity. In addition, they propose a power-assignment algorithm that successfully schedules a network using a poly-logarithmic number of time slots. Approximation scheduling algorithms (see for example [32, 33]) are proposed that compute a feasible solution in polynomial time for the geometric model with worst-case approximation guarantees for arbitrary network topologies when the power levels are constant. In [34] it is shown that solutions with oblivious power assignments (the power level of a node depends only on the transmitter-receiver distance) cannot compete with solutions using possibly different power levels and channels for a network. However, they are capable of achieving nearly the same performance as solutions restricted to symmetric power and channel assignments. For the geometric physical model, the NP-hardness of wireless scheduling without power control is proven in [30] and with power control is proven in [35], given that we know minimum ($P_{min} > 0$) and maximum ($P_{max} < \infty$) transmission power levels.

When considering the abstract physical model less information has to be considered and the values in the gain matrix are not restricted by the topology of the network (not every gain matrix of the abstract model can be expressed as a network, and on the contrary every gain matrix of the geometric model could be a case for the abstract model), i.e., in the geometric physical model we have the advantage of exploiting the geometry of the network in order to check complexity and to design scheduling algorithms. In [36] the NP-hardness of wireless scheduling without power for the abstract physical model is proven and the problem is analytically solved via a Column Generation approach. However, in order to achieve best performance, scheduling and power control should be optimised jointly. This problem is notoriously difficult to solve, even in a centralised manner. In [37], this line of research is followed and the transmission scheduling problem for minimising the total number of slots (channel or time) for variable power levels is formulated. However, in this work they considered non-problem specific analytic solutions that are computationally expensive.

We consider the abstract physical model and the contributions of this paper can be summarised as follows. (i) We first prove that the minimum latency transmission scheduling (MLTS) problem for the abstract model is NP-hard for variable power levels. Contrary to existing approaches and results in the literature, our formulation includes the choice of optimal transmitting powers and arbitrary topology. Furthermore, the generality of the

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abstract physical model implies the complexity of the geometric physical model and completes the theory that the transmission scheduling problem with power control for the physical model in general is NP-hard. This answers the open problem posted in [38].
(ii) We then propose non-trivial upper and lower bounding techniques that, for many cases, are able to find the optimal solution without the need of other optimisation methods.
(iii) For the networks where the optimality gap is not closed, some approaches are discussed that solve the *exact* transmission scheduling problem with power control. While classical techniques are being used from the literature to solve the exact problem, they are tailor-made to the problem and are also combined with our the effective bounding techniques that provide very good upper and lower bounds for the problem, thus achieving enhanced computational performance.

The rest of the paper is organised as follows. In Section 2 the employed system model is described. Section 3 employs the conditions imposed by the model to formulate the transmission scheduling problem for variable power levels as a Mixed Integer Linear Programming (MILP) problem. Section 4 derives some global conditions for feasibility of the network. Section 5 shows that the problem considered is NP-hard, whereas Section 6 firstly presents lower and upper bounding algorithms. Further solution methodologies are discussed, including a column generation approach, specifically designed to find strong bounds for the transmission scheduling problem. Moreover, we implement a B&B algorithm where we enhance the exclusion of non-optimal tree nodes by exploiting the bounds derived in the previous section. The (enhanced) algorithm is found to converge to the optimal solution faster. In Section 7 the validity of our formulation and the performance of our techniques are evaluated. Finally, in Section 9, conclusions are drawn and directions for future work are also given.

2. MODEL

The system model can be divided into two levels: the network as a whole and the channel. Thus, we have the network model and the channel model. The network model concerns the general topology of the nodes and their characteristics. The channel model describes the assessment of the link quality between communication pairs and the interaction between the nodes in the network.

2.1. Network Model

In this study, we consider a network where the links are assumed to be unidirectional and each node is supported by an omnidirectional antenna. For a planar network (easier to visualize without loss of generality), this can be represented by a graph $\mathscr{G} = (\mathscr{N}, \mathscr{L})$, where $\mathscr{N}$ is the set of all nodes and $\mathscr{L}$ is the set of the active links in the network. Each node can be a receiver or a transmitter only at each time instant due to the half-duplex nature of the wireless transceiver. Each transmitter aims to communicate with a single node (receiver) only, which cannot receive from more than one node simultaneously. We denote by $\mathscr{T}$ the set of transmitters and $\mathscr{R}$ the set of receivers in the network.

2.2. Channel Model

The link quality is measured by the Signal-to-Interference-and-Noise-Ratio (SINR). The channel gain on the link between transmitter i and receiver j is denoted by g_{ij} and incorporates the mean path-loss as a function of distance, shadowing and fading, as well as cross-correlations between signature sequences. All the g_{ij}'s are positive and can take values in the range $(0,1]$. Without loss of generality, we assume that the intended receiver of transmitter i is also indexed by i. The power level chosen by transmitter i is denoted by p_i. ν_i denotes the variance of thermal noise at the receiver i, which is assumed to be additive Gaussian noise. The interference power at the i^{th} node, I_i, includes the interference from all the transmitters in the network and the thermal noise, and is given by

$$I_i = \sum_{j \neq i, j \in \mathscr{T}} g_{ji} p_j + \nu_i. \tag{1}$$

Therefore, the SINR at the receiver i is given by

$$\Gamma_i = \frac{g_{ii} p_i}{\sum_{j \neq i, j \in \mathscr{T}} g_{ji} p_j + \nu_i} . \tag{2}$$

The Quality of Service (QoS) is measured in terms of SINR. Hence, independently of nodal distribution and traffic pattern, a transmission from transmitter i to its corresponding receiver is successful (error-free) if the SINR of the receiver is greater or equal to the *capture ratio* γ_i. The value of γ_i depends on the modulation and coding characteristics of the radio. Therefore, we require that

$$\frac{g_{ii} p_i}{\sum_{j \neq i, j \in \mathscr{T}} g_{ji} p_j + \nu_i} \geq \gamma_i. \tag{3}$$

3. PROBLEM FORMULATION

In this section, we present the problem of finding the minimum possible number of time slots (or channels) and the corresponding transmitting powers, such that all communication requests are being processed correctly and Quality-of-Service (QoS) requirements for successful transmissions are satisfied.

Note that to ensure feasibility of our problem we can define the deadline of the network as follows. The maximum number of time slots that may be required is equal to the number of links, $|\mathscr{L}|$. Henceforth, we will assume that the first time slot is at time 1; the latest point in time for which there can be a scheduled transmission is therefore $D = |\mathscr{L}|$. The notation used for the networks in this paper is given below (in Notation 1).

Notation 1 Notation used for the networks:

$\mathcal{N}$	The set of all nodes in the network
$\mathcal{T}$	The set of transmitters in the network
$\mathcal{R}$	The set of receivers in the network
g_{ij}	The channel gain on the link $i \to j$
ν_i	The variance of thermal noise at the receiver i
I_i	The interference power at the i^{th} receiver
Γ_i	The SINR at the i^{th} receiver
γ_i	The capture ratio at the i^{th} receiver
D	The deadline of the network

To formulate the optimisation problem, we define two sets of decision variables, for each transmitter $i \in \mathcal{T}$ and time $t = 1, \ldots, D$; processing-time variables:

$$x_i(t) = \begin{cases} 1, & \text{if transmitter } i \text{ is active at time } t \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

and power level variables: $p_i(t) \in \mathbb{R}_+$.

Since the problem involves both integer and continuous decision variables, the mathematical formulation is classified as a Mixed Integer Program (MIP) and is given in Model 1.

Model 1 Minimum number of time slots

$$\underset{x,p}{\textbf{minimise}} \;\; \tau = \max_{i \in \mathcal{T}} \sum_{t=1}^{D} t x_i(t) \tag{5a}$$

subject to

$$\sum_{t=0}^{D} x_i(t) \geq 1 \; \forall i \in \mathcal{T}, \tag{5b}$$

$$x_i(t) = 0 \Rightarrow p_i(t) = 0 \; \forall i \in \mathcal{T},\, t = 1, \ldots, D, \tag{5c}$$

$$x_i(t) = 1 \Rightarrow g_{ii} p_i(t) \geq \gamma_i \left(\sum_{j \in \mathcal{T}, j \neq i} g_{ji} p_j(t) + \nu_i \right) \tag{5d}$$

$$\forall i \in \mathcal{T},\, t = 1, \ldots, D,$$

$$x_i(t) \in \{0, 1\} \; \forall i \in \mathcal{T},\, t = 1, \ldots, D, \tag{5e}$$

$$p_i(t) \in \mathbb{R}_+ \; \forall i \in \mathcal{T},\, t = 1, \ldots, D. \tag{5f}$$

Objective (5a) minimises the number of time slots needed to schedule all the transmitters in the network. For every transmitter $i \in \mathcal{T}$, $\sum_{t=1}^{D} t x_i(t)$ is equal to the scheduling time, since only one of $x_i(1), x_i(2), \ldots, x_i(D)$ will be equal to 1. We define as τ the latest scheduling time for a transmission. We can linearise the objective by requiring τ to be larger than or equal to all of the pairs' scheduling time, i.e., $\tau \geq \sum_{t=1}^{D} t x_i(t)$ for all $i \in \mathcal{T}$. Constraint (5b) ensures that each link in the network is processed at least once in the schedule. Note that there is always an optimal schedule in which each link is processed only once. Constraint (5c) makes sure that if a pair is not processed at a specific time slot, then the power level of the corresponding transmitter is 0 at that time slot. The QoS conditions are guaranteed by constraint (5d). The constraint only affects the optimisation if $x_i(t)$ takes the value 1. Finally, the last two constraints (5e) and (5f) define the admissible values for the decision variables.

4. PRELIMINARIES

Inequality (3) depicts the QoS requirement of a communication pair i while transmission takes place. After manipulation it becomes equivalent to the following

$$p_i \geq \gamma_i \left(\sum_{j \neq i, j \in \mathcal{T}} \frac{g_{ji}}{g_{ii}} p_j + \frac{\nu_i}{g_{ii}} \right). \tag{6}$$

In matrix form, for a network consisting of n communication pairs, this can be written as

$$\mathbf{p} \succeq \Gamma G \mathbf{p} + \boldsymbol{\eta} \tag{7}$$

where $\Gamma = \operatorname{diag}(\gamma_i)$, $\mathbf{p} = \begin{pmatrix} p_1 & p_2 & \ldots & p_n \end{pmatrix}^T$, $\eta_i = \frac{\gamma_i \nu_i}{g_{ii}}$ and

$$G_{ij} = \begin{cases} 0 & , \text{if } i = j, \\ \frac{g_{ji}}{g_{ii}} & , \text{if } i \neq j. \end{cases}$$

Let

$$C = \Gamma G, \tag{8}$$

so that (7) can be written as

$$(I - C)\mathbf{p} \succeq \boldsymbol{\eta} \tag{9}$$

Matrix C has strictly positive off-diagonal elements which implies that it is irreducible, since we are not considering totally isolated groups of links that do not interact with each other. By the Perron-Frobenius theorem [39], we have that the spectral radius of the matrix C is a simple eigenvalue, while the corresponding eigenvector is positive component-wise. A necessary and sufficient condition for the existence of a nonnegative solution to inequality (9) for every positive vector $\boldsymbol{\eta}$ is that $(I - C)^{-1}$ exists and is nonnegative. However, $(I - C)^{-1} \succeq 0$ if and only if $\rho(C) < 1$ [39] (where $\rho(C)$ denotes the spectral radius of C), or, equivalently, $(C - I)$ is Hurwitz (since $(C - I)$ is Metzler), see [40].

Therefore, the necessary and sufficient condition for (7) to have a positive solution $\mathbf{p}^*$ for a positive vector $\boldsymbol{\eta}$ (i.e., there exists a set of powers such that all the senders can transmit simultaneously and still meet their QoS requirements (minimum SINR for successful reception) is that the Perron-Frobenius eigenvalue of the matrix C is less than 1.

5. COMPUTATIONAL COMPLEXITY

Theorem 1
Problem (5) is NP-hard.

Proof
The statement of Theorem 1 is equivalent to saying: deciding whether the optimal value of (5) does not exceed a

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given value $\kappa \in \mathbb{N}$ is NP-hard. We construct a polynomial-time reduction of the Graph Colouring problem, which is well known to be NP-hard [41]. Given an undirected graph $G = (V,E)$ with nodes $V = \{1,\dots,n\}$ and edges

$$E \subseteq \{\{i,j\} : i,j \in V, i \neq j\},$$

as well as a scalar $k \in \mathbb{N}$, the Graph colouring problem asks whether there is an assignment $f : V \mapsto \{1,\dots,k\}$ of nodes to k colours such that $f(i) \neq f(j)$ for all $\{i,j\} \in E$, that is, neighbouring nodes must have different colours. Our reduction takes as input a Graph colouring instance and generates an instance of (5) such that the optimal value of problem (5) does not exceed $\kappa = k$ if and only if the answer to the Graph colouring problem is affirmative. Towards this end, we set $\mathscr{T} := V$, $D := |V| = n$, $\mathscr{L} := E$, $\gamma = (1,\dots,1)^\top$, $v_i = 0\ \forall\, i \in \mathscr{R}$ and

$$g_{ij} := \begin{cases} 1 & \text{if } \{i,j\} \in E, \\ 1/2 & \text{if } i = j, \\ 1/(2n) & \text{otherwise.} \end{cases}$$

The size of this reduction is polynomial in the size of the Graph colouring instance. Hence, if we show that the optimal value of (5) does not exceed κ if and only if the answer to the Graph colouring instance is affirmative, we have proven that the solution of (5) is NP-hard. We proceed in two steps. Firstly, we show that if there is a Graph Colouring that uses ζ colours, then the optimal value of (5) is smaller or equal to ζ. Secondly, we show that if there is a feasible solution for (5) of value ζ, then we can construct an admissible ζ-colouring for the Graph Colouring instance. The assertion follows from the combination of both arguments and the fact that we consider a minimisation objective.

As for the first step, assume that there exists a colouring $f : V \mapsto \{1,\dots,\zeta\}$. Given this colouring, we construct a feasible solution (x,p) of objective value ζ. Towards this end, we set $x_i(t) := 1$ if $f(i) = t$ and $x_i(t) := 0$ otherwise, $\forall\, i \in \mathscr{T}$. Likewise, set $p_i(t) := \hat{p}_i$ ($\hat{p}_i \in \mathbb{R}_+$) if $f(i) = t$ and $p_i(t) := 0$ otherwise, for all $i \in \mathscr{T}$ and $t = 1,\dots,D$. By construction, constraints (5b), (5c), (5e) and (5f) are satisfied, for any value $\hat{p}_i$, $\hat{p}_i \in \mathbb{R}_+$. For $i \in \mathscr{T}$ and $t \in \{1,\dots,D\}$ with $x_i(t) = 1$, constraint (5d) requires that

$$p_i(t) \geq \left(2 \sum_{\{i,j\}\in E} p_j(t)\right) + \frac{1}{n} \sum_{\substack{j\in V,\, j\neq i,\\ \{i,j\}\notin E}} p_j(t).$$

Since f constitutes a valid colouring, the first term on the right hand-side must evaluate to zero, because otherwise, the spectral radius ρ of the matrix that is constituted by $\{i,j\} \in E$ is greater than 1 and hence the network would be infeasible, as described in Section 4. On the other hand, when the first term of the right hand-side is zero, the second term fulfills the inequality, since

$$\sum_{\substack{j\in V,\, j\neq i,\\ \{i,j\}\notin E}} \frac{1}{n} < 1 \Rightarrow \|C\|_\infty < 1,$$

and hence, $\rho(C) < 1$, where C consists of $i,j \in V$, $j \neq i$, $\{i,j\} \notin E$. We conclude that constraint (5d) is also satisfied by our choice of x and $p \in \mathbb{R}_+$. Note that the objective function (5a) evaluates to ζ for the constructed solution (x,p), which implies that the optimal value of (5) must be smaller or equal to ζ.

In the second step, we use a feasible solution to (5) with objective value ζ to construct a valid colouring of the graph G with at most ζ colours. Assume that we have a feasible solution (x,p) for problem (5) with objective value ζ. Since we consider a minimisation problem, without loss of generality we can assume that $\sum_{t=1}^{D} x_i(t) = 1$. Hence, we obtain a function if we set $f(i) := t$ if and only if $x_i(t) = 1$. Furthermore, since the objective value (5a) of (x,p) is ζ, the range of f is limited to $\{1,\dots,\zeta\}$. We now show that f constitutes a valid colouring of graph G, that is, $f(i) \neq f(j)$ for all $\{i,j\} \in E$. Assume to the contrary that there is $\{\hat{\imath},\hat{\jmath}\} \in E$ with $f(\hat{\imath}) = f(\hat{\jmath}) = \hat{t}$. In this case, (x,p) must satisfy the constraints

$$p_{\hat{\imath}}(\hat{t}) \geq 2 \sum_{\{\hat{\imath},j\}\in E} p_j(\hat{t}) + \frac{1}{n} \sum_{\substack{j\in V,\, j\neq \hat{\imath}\\ \{\hat{\imath},j\}\notin E}} p_j(\hat{t}) \geq 2 p_{\hat{\jmath}}(\hat{t})$$

and

$$p_{\hat{\jmath}}(\hat{t}) \geq 2 \sum_{\{\hat{\jmath},i\}\in E} p_i(\hat{t}) + \frac{1}{n} \sum_{\substack{i\in V,\, \hat{\jmath}\neq i\\ \{\hat{\jmath},i\}\notin E}} p_i(\hat{t}) \geq 2 p_{\hat{\imath}}(\hat{t}),$$

which contradicts the assumption that (x,p) is feasible for problem (5). We conclude that the constructed function f indeed constitutes a valid ζ-colouring of the graph G. As a result, problem (5) is NP-hard. □

6. SOLUTION APPROACHES

In this section, we propose solution approaches for the wireless scheduling problem with power control (Model 1). In particular, we firstly describe lower (LB) and upper bounding (UB) techniques. In addition, we describe solution methodologies designed to converge to the optimal solution for the cases when the optimality gap using the bounds is not closed. As aforementioned, these algorithms make use of the derived bounds, thus enhancing their performance in terms of computational efficiency.

6.1. Lower and Upper bounds

6.1.1. Lower Bounds

Algorithm 1 presents a lower bounding technique that is based on a sorting process whose metric is the number of nodes that each node cannot be simultaneously processed with. The algorithm begins with setting values ε_{ij} equal to 1 if nodes i and j cannot be simultaneously processed, 0 otherwise. We can check if two nodes can be processed simultaneously by isolating them as a network and by checking its spectral radius. We construct set $CL = \mathscr{N}$

and sort it in decreasing order according to $\sum_{j\in\mathscr{T}} \varepsilon_{ij}$, i.e. the number of the nodes which node i cannot be simultaneously processed with. The first member of set CL is deemed as the first slot leader. Then, each node in position 2 onwards is checked with all the nodes in previous positions. If a node *can* be simultaneously present in the same time slot with any one of the checked nodes, then the node is removed from set CL. Otherwise, it becomes a slot leader. The number of slot leaders at the end of this operation is the obtained lower bound LB. Note that due to the fact that we check if two nodes can be in the same slot in each case, and not the whole set of nodes assigned to the specific slot, this scheme will allow more nodes in the slot than it would otherwise admit (the final selection is therefore not necessarily feasible), and hence less slots will be required in total. That is why this methodology constitutes a lower bounding technique.

Algorithm 1 Lower bounding technique LB

initialise
 Set $\varepsilon_{ij} \leftarrow \begin{cases} 1, \text{if } \rho([0,c_{ij};c_{ji},0]) > 1 \\ 0, \text{ otherwise} \end{cases}$ $\forall i,j \in \mathscr{T}$ and set $CL_i = i$. Resort CL in decreasing order of $\sum_{j\in\mathscr{T}} \varepsilon_{ij}$.
 Set $k \leftarrow 2$.

while $k \leq |CL|$ **do**
  $i \leftarrow CL_k$.
  for $m = 1$ to $k-1$ **do**
   $j \leftarrow CL_m$.
   if $\rho([0,c_{ij};c_{ji},0]) < 1$ **then**
    Remove i from CL.
    $k \leftarrow k-1$
    Exit FOR loop.
   end if
  end for
  $k \leftarrow k+1$
end while
Set $LB \leftarrow |CL|$.
return LB

We also present a variation of the lower bound LB, referred to as LB', which includes two additional steps to Algorithm 1. The first addition to the algorithm is performed at the end of each iteration of the WHILE loop. In particular, we update values ε_{ij} for $i,j \in CL$ and consequently update and resort set CL. The final addition is a checking process which may increase the lower bound by 1. To explain this, Algorithm 1 implicitly assumes that a number of nodes can be simultaneously executed in the same time slot if they at least can be simultaneously executed with the so-called slot leader. In polynomial time, we can check whether the nodes implicitly assumed to be executed with the **final** slot leader cannot be pairwise simultaneously executed. If we can find such "infeasible" pairs, then LB' is equal to $LB+1$. To clarify this, we note that the members of the infeasible pair cannot be both present in the final slot. Furthermore, neither of the members can be scheduled at any previous time slots (by construction of the lower bound). Hence, one of the members of the pair must be scheduled in a new time slot.

6.1.2. Upper Bound

In the literature, upper bounding techniques are described as approximation scheduling methods (see for example [32, 33, 35] and references therein). We describe two non-trivial upper bound methods that, when combined with strong lower bounding techniques, are capable of closing the optimality gap, hence finding the optimal solution efficiently. We first describe a simple yet effective heuristic, based on a priority scheduling policy. The derived solution value, referred to as UB, serves also as a cut-off value in the B&B approach described in the next subsection. The basic idea of the policy is to keep adding new transmissions at the current time slot according to a priority criterion, until no more transmissions can be scheduled without violating the SINR constraints. In such a case, the next time-slot is considered, and the process is repeated for all the remaining unscheduled transmission pairs. Note that for each node considered for a time slot, the spectral radius of the matrix that constitutes the network is calculated, which takes time $O(n^3)$. In our algorithm, the priority value of pair $i \in \mathscr{T}$ is found using

$$R_i = \frac{\nu_i \gamma_i}{g_{ii}}, \tag{10}$$

which effectively represents the power that transmitter i produces in a time slot when it is the only active transmitter. The complete heuristic algorithm is described in Algorithm 2 below. Note that set A (in the description below) contains all transmission pairs in decreasing order of their priorities.

Algorithm 2 Upper bounding technique

initialise
  $S = \emptyset$
  $A = \{i_1,\ldots,i_{|\mathscr{T}|} | i_k \in \mathscr{T}, \forall k \leq |\mathscr{T}|, R_{i_{k_1}} \geq R_{i_{k_2}} \text{ if } k_1 \leq k_2\}$
  $UB \leftarrow 0$

while $S \neq A$ **do**
  $set = \emptyset$
  for $j \in A$ **do**
   if $j \cup set$ satisfies SINR constraints **then**
    $set \leftarrow set \cup \{j\}$
    $A \leftarrow A \setminus \{j\}$
    $S \leftarrow S \cup \{j\}$
   end if
  end for
  $UB \leftarrow UB + 1$
end while

return UB

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Note that, in this case, since not the optimal set of pairs is chosen to be admitted in each slot, the allocation of the pairs will be suboptimal and the number of slots required will be an upper bound for the minimum number of slots. For constant noise, the priority value is equivalent to the sorting in [32] of the links by nondecreasing order of length. However, apart from taking into account that thermal noise could differ at the receivers, our approach calculates the spectral radius of the matrix each time a node is admitted in a network, thus allowing for variable power; hence, it is less conservative than the *affectedness* and *affectance* used in [32] and [33], respectively, since it is essentially a metric larger than $\|C\|_\infty$, which is already conservative [42].

While UB produces very good upper bounds, we propose an additional upper bounding algorithm, called UB' and described in Algorithm 3, that in some instances (but not all) outperforms UB. This approach chooses the order by which the nodes enter a network by the interference they experience. The algorithm is as follows. The first step of the algorithm is to calculate the sum of each row and column in matrix C as given by (8). The sum of each column represents the interference *caused by* the particular node while the sum of each row represents the interference *experienced by* the particular node. The algorithm chooses the node with maximum caused interference and places it in a time slot. The algorithm then checks the remaining nodes in decreasing order of the interference experienced. If a node is feasible with the members in a time slot, then it is included in the same set. Otherwise, it is placed in a new time slot.

Algorithm 3 Upper bounding technique

initialise
Set $sumrows_i \leftarrow \sum_{j \in \mathscr{T}} c_{ji}$ and $sumcols_i \leftarrow \sum_{j \in \mathscr{T}} c_{ij}$. Find $icol \in \mathscr{T}$ such that $sumcols_{icol}$ is maximum.
Set $k \leftarrow 1$ and $S_k \leftarrow \{icol\}$. Update $\mathscr{T} \leftarrow \mathscr{T} \setminus \{icol\}$.

while $\mathscr{T} \neq \emptyset$ **do**
  Find $irow \in \mathscr{T}$ such that $sumrows_{irow}$ is maximum.
  for $m = 1$ to k **do**
    if $irow \cup S_m$ satisfies SINR constraints **then**
      $S_m \leftarrow S_m \cup \{irow\}$
      Exit FOR loop.
    end if
  end for
  if $irow$ could not be placed in any of the sets S **then**
    $k \leftarrow k+1$, $S_k \leftarrow \{irow\}$
  end if
  $\mathscr{T} \leftarrow \mathscr{T} \setminus \{irow\}$
end while

Set $UB \leftarrow k$.
return UB

We also present a variant of the upper bounding technique given in Algorithm 3 which includes an additional step. In particular, we add a resorting process at the end of each iteration of the WHILE loop. The process finds the minimum total power emitted by the nodes in each set S such that the SINR constraints are satisfied. The sets are then resorted in increasing order of the total power. The idea is to add more members to the sets with lower power levels first. The resulting upper bound is referred to as UB''. Our upper bounding techniques are compared is Section 7 with *ApproxLogN* algorithm proposed in [32], which is considered the current state-of-the-art.

6.1.3. Column Generation method

We also describe a column generation technique, based on an alternative formulation of the original problem, which is capable of providing both a lower bound and an upper bound. It is important to note that the main contribution of the method is the fact that it obtains stronger lower bounds. The new formulation uses an explicit representation of *feasible* sets of transmission pairs. A set of transmission pairs $s \subseteq \mathscr{T}$ is said to be feasible if the simultaneous execution of all the pairs in the set does not violate the SINR constraints (5d).

The new set covering formulation, given in Model 2 incorporates the complete set of feasible sets of transmission pairs, S. A binary decision variable is associated to each feasible sequence $s \in S$, defined as

$$\vartheta_s = \begin{cases} 1, & \text{if set } s \text{ is used in the optimal solution;} \\ 0, & \text{otherwise.} \end{cases}$$

Model 2 Set covering formulation

$$\textbf{minimise} \sum_{s \in S} \vartheta_s \tag{11a}$$

subject to

$$\sum_{s \in S: i \in s} \vartheta_s \geq 1 \qquad \forall i \in \mathscr{T} \tag{11b}$$

$$\vartheta_s \in \{0,1\} \qquad \forall s \in S \tag{11c}$$

The objective is to minimise the number of sets that are required in the optimal solution. Constraints (11b) ensure that the solution includes at least one set for each pair $i \in \mathscr{T}$ and constraints (11c) define the allowable range of values for the decision variables.

In order to efficiently manage the complexity of the exponential number of variables, we solve the continuous relaxation of Model 2 (master problem) via a column generation scheme. The master problem is given by equations (11a), (11b) and $0 \leqslant \vartheta_s \leqslant 1 \; \forall s \in S$. The optimal solution is given as $LBcg$, a valid lower bound to Model 1.

The master problem is initially solved using $S' \subseteq S$, an initial subset of set S, and the dual values π_i^*, associated to constraints (11b), are found. New variables (sequences) are generated one-by-one by finding sets $s^* \subseteq S$ such that the dual constraint

$$\sum_{i \in s^*} \pi_i^* \leqslant 1 \tag{12}$$

is violated, i.e. such that

$$\sum_{i \in I: i \in s^*} \pi_i^* > 1 + \varepsilon.$$

Set s^* is found by solving Model 3, the sub-problem, which finds a feasible set of transmission pairs of maximum violation. For the mathematical formulation, we define binary decision variables:

$$\zeta_i = \begin{cases} 1, & \text{if pair } i \in \mathscr{T} \text{ is present in the set } s^* \\ 0, & \text{otherwise} \end{cases}$$

and decision variables $\mu_i \in \mathbb{R}^+$, the power level of pair $i \in \mathscr{T}$.

Model 3 Sub-problem

$$\textbf{maximize} \sum_{i \in \mathscr{T}} \pi_i^* \zeta_i \tag{13a}$$

subject to

$$\zeta_i = 0 \Rightarrow \mu_i = 0 \qquad \forall i \in \mathscr{T} \tag{13b}$$

$$\zeta_i = 1 \Rightarrow g_{ii}\mu_i \geq \gamma_i \Big(\sum_{j \in \mathscr{T}, j \neq i} g_{ji}\mu_j + \nu_i \Big) \qquad \forall i \in \mathscr{T} \tag{13c}$$

$$\zeta_i \in \{0,1\} \qquad \forall i \in \mathscr{T} \tag{13d}$$

$$\mu_i \in \mathbb{R}^+ \qquad \forall i \in \mathscr{T} \tag{13e}$$

The objective function (13a) aims at finding the maximum violation associated to a feasible set of transmission pairs. Constraints (13b) and (13c) ensure that if a transmission pair is not present in the set, then its associated power level is zero and for all pairs in the set, the SINR conditions are satisfied, respectively. Finally, constraints (13d) and (13e) define the allowable values for the decision variables.

The master problem and sub-problem are solved repeatedly until no more sequences violating constraint (12) can be found. Significant improvements in terms of the speed of the column generation methodology are achieved by constructing and inserting a population of feasible sequences before the column generation is invoked. The population includes all the sequences which include up to two pairs, found using complete enumeration. We also include a subset of the sequences with at least three pairs, constructed using the heuristic approach described in Section 6.1.2.

Note that, by the end of the column generation approach a fractional value for the master problem may be found. In such a case, the value can be rounded up to the nearest integer, thus guaranteeing feasibility of the solution.

Finally, at the end of the column generation approach, a number of feasible sets of transmission pairs is found, including the ones which were enumerated prior to the column generation as well as the ones found using the sub-problem (Model 3). Let $\bar{S}$ be the collection of all these sets. If we solve Model 2 by substituting S with $\bar{S} \subseteq S$, then the solution value found will be an upper bound to the original problem. This upper bound is denoted by $UBcg$.

6.1.4. Branch & Bound Approach

For the purposes of this paper, we implement B&B approaches, which at the initialization stage set their lower bound as the maximum of lower bounds LB and LB' and the upper bound (cut-off value) as the minimum of upper bounds UB, UB' and UB''. It is possible also to add *feasible cuts*, i.e. constraints which aim to exclude any solutions which we are certain to be non-optimal. For example, we may add constraints which force infeasibility on the solutions found with our upper bound techniques. As a result, the B&B algorithm will try to find the best solution not equal to the latter solutions, and if it cannot, then the excluded solutions were optimal. We classify such cuts as "pairwise" and "all", denoting feasibility cuts that disallow the simultaneous execution of pairs of nodes and of a number of nodes greater or equal to 2, respectively. For "pairwise" cuts, we can find the pairs using complete enumeration (as we did in Section 6.1.1). For "all" feasible cuts, we include the "pairwise" cuts as well as the sets of solutions derived from the upper bounding techniques. These are not strictly speaking "infeasible", but we do not lose optimality by excluding them from the search since their solution value has been noted via the "cut-off" value. We refer to the B&B approaches $BB_{paircuts}$ and BB_{cuts}, as the B&B algorithms incorporating "pairwise" and "all" cuts, respectively. For benchmarking purposes, we also use another B&B approach, referred to as BB, which is generic as it only implements CPLEX's default settings (no bounds are incorporated).

7. PERFORMANCE EVALUATION

All the algorithms as well as the lower and upper bounding techniques described, have been implemented in Microsoft Visual Studio 2005 C++ using CPLEX v12.1 and run on an Intel Core 2 computer, with 2.5GHz processor and 3.5GB of RAM.

Throughout the paper, we set $\gamma_i = 3$ and $\nu_i = 0.04$ *mWatts*. For each example, the selected number of nodes is uniformly and independently distributed on a square of side $100m$, making sure that no two nodes have distance between them less than $1m$. The links are constructed with a neighbor algorithm, i.e., each transmitting node will choose a neighboring node as its receiver. Then, the channel gains g_{ji} are obtained by considering distance attenuation only, i.e., $g_{ji} = (d_0/d)^{\alpha}$, where $d_0 = 1m$ and $\alpha = 4$.

The algorithm begins with the computation of a lower bound (LB) and an upper bound (UB) according to the

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techniques described in Section 6.1.1 and the techniques described in 6.1.2, respectively. It is evident that if LB has the same value as UB, any optimisation technique is redundant; the heuristic solution would be optimal for the transmission scheduling problem. For the networks for which the calculated upper and lower bounds have different values (e.g., if upper bound= 5 and lower bound= 3) we use optimisation techniques in order to close the gap and hence, obtain the optimal solution. At this stage, the initial pairwise cuts are found and added to the problem.

After determining the bounds, if the optimal solution is not found, we use Branch & Bound (B&B) which utilises the calculated bounds in order to derive the optimal solution faster. The apparent gain from determining as tight bounds as possible is the decrease in CPU time required by the B&B technique. Apart from the optimisation techniques, a column generation (CG) optimisation approach was designed in order to find stronger bounds for the transmission scheduling problem. The performance of CG was then compared with the other methods.

The performance of our algorithms was evaluated for 60 different networks of six different sizes: 10, 20, 30, 40, 50 and 60 pairs of nodes. For each size we investigated 10 different networks. The performance of the algorithms is evaluated on three aspects: (a) the algorithms' speed, (b) their scalability, and (c) their success at finding an optimal solution. Most bounding algorithms considered in the literature, consider only upper bounding techniques and therefore, a fair comparison can be done only for the upper bounding methods proposed.

Figure 1 corresponds to the computational time for the LB and UB techniques for the networks considered of different sizes. The figure demonstrates that the computation time for both the lower and upper bounds is low for small networks and also it scales linearly with the number of communication pairs considered in the network. As a result, the algorithms succeed in having a low computational cost and good scalability properties.

In Figure 2 we compare the upper and lower bounds with the optimal value for each network considered, in order to get insight as to whether the non-trivial bounds designed perform well against the optimal values. For the figure, we can easily observe that for small networks the bounding algorithms provide bounds which converge to the optimal value fast and with high probability, whereas for larger networks the bounds are not tight, but seem to remain close to the optimal value.

In Figure 3 it can be deduced that the UB algorithm produces slightly better results than the LB, since it matches the optimal solution more often than the LB algorithm but they both remain close to the optimal value as the number of communication pairs increases. Our UB algorithm also outperforms *ApproxLogN* [32].

For the networks for which the Bounds algorithms (Sections 6.1.1 and 6.1.2) did not converge to the optimal value we have used optimisation algorithms. From a total of 60 networks, the Bounds algorithms (sections 6.1.1 and

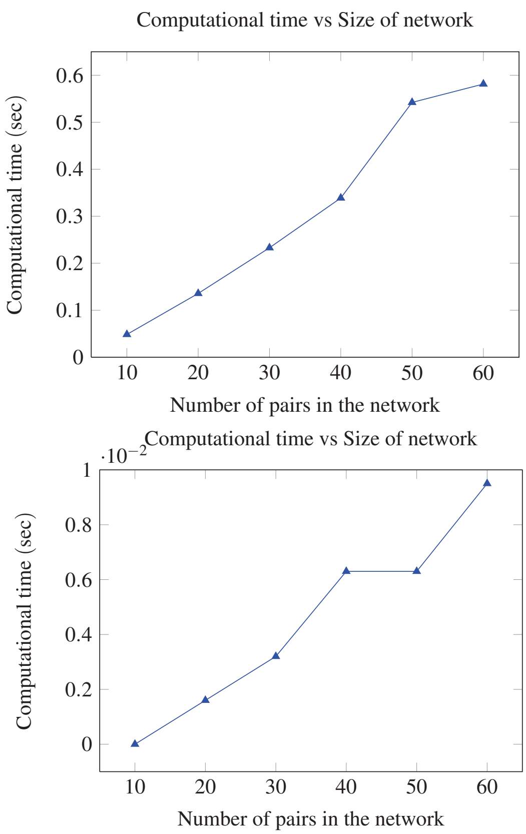

Figure 1. Average CPU times for the UB and LB algorithms. It is easy to see that the computational complexity of the algorithms scales linearly with the number of communication pairs in the network.

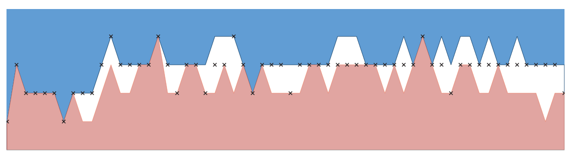

Figure 2. LB, UB and optimal values for all networks considered. The networks from left to right are put in order such that each of the first 10 networks consists of 10 communications pairs. Then, every 10 networks the number of communication pairs increases by 10.

6.1.2) did not converge to the optimal value for 35 of them. It is remarkable to note that the lower bound, which was always the weakest bound, has comparable results to the upper bound; that illustrates that the derived non-trivial lower bound is tighter than existing results.

As shown in Table I, the computational cost for both lower and upper bounds is very small. When these bounds are incorporated into the B&B algorithm we have significant improvements in the performance of the algorithms and this will be shown next in Section 7.1.

Table I. Average calculation time and total calculation time vs optimality

Number of Nodes	Average calculation time for LB (sec)	Average calculation time for UB (sec)	Average normalised maximum calculation time $[\frac{\max(t_{LB},t_{UB})}{N}]$	Optimality for LB (%)	Optimality for UB (%)	Optimality (%)
10	0	0.0733	0.0073	80	100	80
20	0.0016	0.1357	0.0068	40	90	30
30	0.0032	0.2329	0.0078	60	60	40
40	0.0063	0.3388	0.0085	80	60	40
50	0.0063	0.5422	0.0108	70	50	40
60	0.0095	0.5813	0.0097	20	70	10
Averages:	0.0068	0.317	0.007	58.33	71.67	40.00

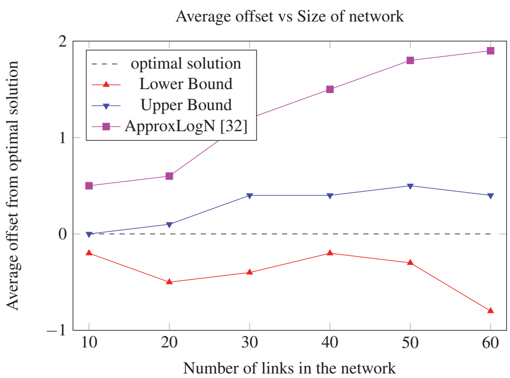

Figure 3. Average distance of bounds from the optimal value for networks of size 10, 20, 30 40, 50 and 60 communication pairs.

7.1. Evaluation of the CG and B&B techniques

For the 35 networks whose optimal solutions were not found using the bounds, the CG method converged for 9 networks (25.7%). Figure 4 compares the performance (in CPU time) of the CG and B&B approaches for these 9 networks.

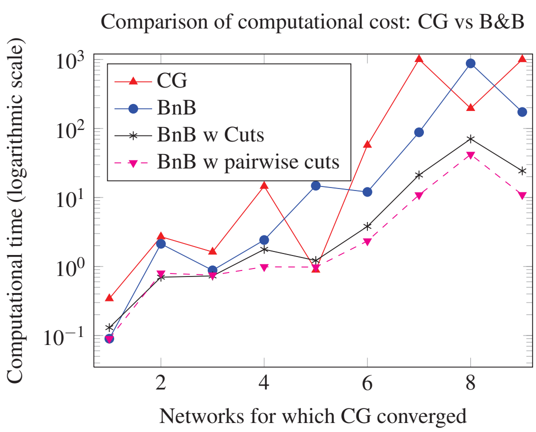

Figure 4. Performance of CG with respect to the B&B algorithms for the 9 networks that CG converged.

We can conclude that although CG is designed as a lower bounding approach, in fact, all B&B variations are faster. The only exception are two networks, where CG performs slightly better than most of the B&B algorithms, except from the $BB_{paircuts}$. Given CG's low optimality performance and its underperformance time-wise, we can infer that the scenarios where CG would be more appropriate than B&B cuts or B&B with pairwise cuts are limited and negligible. Due to CG's poor performance we conclude that it is not an attractive approach for the solution of the scheduling problem.

8. DISCUSSION

In this paper, we studied the MLTS problem when power control can take place, i.e., the problem of minimising the number of time-slots required for scheduling all the wireless nodes in a given network for the abstract physical model when power control is allowed. The contributions of this paper are as follows.

(a) We proved that this problem is NP-hard. Contrary to existing approaches and results, our formulation includes the choice of optimal transmitting powers and arbitrary topology. Furthermore, the generality of the abstract physical model implies the complexity of the geometric physical model and completes the theory that the minimum latency transmission scheduling problem with power control for the physical model in general is NP-hard. [30] and [35] prove NP-hardness for the geometric physical model without and with power control, respectively, and not the abstract physical model. For the abstract physical model NP-hardness is proven in [36], but for constant power levels. The NP-hardness proof for the abstract physical model with power control has been missing (the open problem posted in [38]). Through our proof the conditions for which this problem becomes equivalent to graph colouring problem are presented. This would be helpful to know in scenarios in which we are asked to construct the network and hence, we will be able to avoid conditions that would make the computational complexity large.

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(b) We have developed non-trivial lower and upper bounds that can, in many instances, provide the optimal solution to the problem. In our simulations, it is illustrated that the upper bound is better in general than the lower bound and outperforms the current state-of-the art approximation algorithm (*ApproxLogN* [32]). The lower bound was shown to be better than many other approaches (e.g., relaxations of optimisation formulations of the problem), but still remains an open problem.
(c) Even if the bounds do not converge to the optimal solution, then when incorporated into the MILP formulation, it provides a considerable improvement in terms of computational time, as shown in the performance evaluation. As a result, the combined methodology scales to problems of nontrivial size. The column generation approach (also appearing in many optimisation problems) has poor performance when compared to our problem specific B&B approach and hence, it is not an attractive approach for the solution of the scheduling problem. Other approaches, such as Cutting Plane, have been investigated (see for example [1]) but they were proved inferior to B&B approaches.

9. CONCLUSIONS AND FUTURE DIRECTIONS

The minimum latency transmission scheduling problem in wireless networks for an intrinsically global model such as the abstract physical model with power control is NP-hard; it is thus unlikely to admit a polynomial-time optimal solution. The NP-hardness of the scheduling problem with power control for the abstract physical model, due to its generality, implies the NP-hardness for the geometric SINR model also. Therefore, the emphasis now is towards techniques that can provide strong, non-trivial lower and upper bounds for enhanced computational performance of analytical methods. To this end, we developed efficient bounding techniques that find good upper and lower bounds to the transmission scheduling problem. Further, we incorporated these bounds into a B&B implementation, showing that we are able to scale to problems of non-trivial size. Both the exact and heuristic approaches are useful in deriving the optimal solution value quickly, as well as providing feasible solutions of known quality in case the optimal solution is unknown. The significance of these results is three-fold. On the one hand, the problem of transmission scheduling where transmitters are able to adjust their power levels to fully benefit from spatial reuse, has been formulated and solved more efficiently with the aid of effective bounding techniques. On the other hand, the results are of practical importance in the presence of a central controller; the controller is able to make the calculations and disseminate the information to the rest of the network (for example in cellular networks where the base station can act as a centralised agent). Finally, the solution constitutes an important benchmark when evaluating approximation algorithms or distributed algorithms for scheduling when knowledge of the whole network is unavailable.

Current research focuses on finding bounds that guarantee that the error lies within a maximum distance from the optimum solution, or, even prove hardness-of-approximation for an arbitrary gain matrix, i.e., that no reasonable approximation algorithms can be developed for this problem. In addition, future work includes the implementation of the suggested algorithms on a Field-Programmable Gate Array (FPGA) whose computational performance will be compared with those coded on a PC using CPLEX. Furthermore, the development of approximation algorithms for the transmission scheduling problem for the abstract physical model is part of ongoing research. Finally, a distributed algorithm poses a very challenging task and still remains an open problem.

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