Precoding Decision for Full-Duplex X-Relay Channel with Decode-and-Forward

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Abstract—In this paper, we study a simple X-relay configuration where the shared relay operates in full-duplex (FD) mode. The relay node may have limited spatial degrees of freedom, and as a result, it may not be able to handle both the loop interference and the multiuser interference. Hence, a decision on the precoding scheme is necessitated. It is often the case that the relay does not have the option of real-time switching between different precoding schemes, either due to hardware limitations of the relay or increased complexity of the problem. Hence, we investigate a "static" precoding decision where the relay node decides on its precoding scheme based only on statistical knowledge of the channel conditions. To perform this decision, the behavior of the system is formulated as a Markov chain and the outage probability of the system is derived in a closed-form with the precoding decision as a parameter. The outage probability is minimized by optimally choosing the precoding scheme, using easily verifiable conditions on the statistical knowledge of the channel conditions. Simulations validate the investigated scheme.

Index Terms—X-relay channel, Decode-and-Forward, precoding design, full-duplex relaying, Markov chains, outage probability.

I. INTRODUCTION

Full-duplex (FD) relaying resolves the problem of bandwidth loss associated with half-duplex (HD), as it completes the relaying transmission in a single channel. Nevertheless, FD suffers from the interference caused from the relay output to the relay input (see, for example, [1], [2]), the so-called *loop interference* (LI). Most of the work done on FD relaying deals with the mitigation of the LI that affects the relay's input. From a hardware perspective, the investigation of efficient analog and/or digital interference cancelation (IC) techniques that suppress the LI is an emergent topic (see [3], [4] and references therein). However, the application of IC techniques is not sufficient in order to completely remove the LI, and thus residual LI components remain after the IC process [1]. For example, it is illustrated in [2] that time-domain cancellation suffers from residual interference due to the transmit signal noise. In most of the cases, this residual LI scales with the transmitted relays's power and affects the system's performance by resulting in a zero-diversity order [5], [6].

In order to further mitigate the effects of LI, signal processing techniques, such as precoding design, have been investigated in the literature. For example, Riihonen *et al.* [2] analyze a broad range of IC techniques (natural isolation, time-domain cancellation and spatial suppression) and deal with the design of zero-forcing (ZF)-based reception/transmission

filters at the relay node in order to eliminate LI. The work in [7] also investigates the filter design at the relay node by using the maximization of the ratio between the power of the useful signal to the LI as a criterion. In [8], the authors do not limit the filter design on the LI and optimize the precoding weights by considering the end-to-end performance of the system. On the other hand, recent studies on the precoding design for FD relaying take also into account the source node and investigate space-time coding techniques that ensure diversity [9], [10].

In this paper, we study the precoding problem for an X-relay configuration, where a shared FD relay helps two sources to transmit their data to two destinations; the X-relay channel is considered as a basic network structure and has been extensively studied in the HD context (see, for example, [11], [12] and references therein). Specifically, we consider a scenario with limited spatial degrees at the relay node as well as partial instantaneous channel state information (CSI). In addition, the relay node cannot perform real-time switchings of the precoding matrix. These limitations (forced by strict hardware/complexity constraints) require that the FD relay node cannot mitigate both the LI and the *multi-user interference* (MUI), as it has not the required number of antennas and in addition, it cannot track both and switch in real-time between instantaneous LI and MUI channels. Based on these design constraints, we investigate a static precoding decision that takes into account only a statistical knowledge of the channel conditions. The proposed methodology formulates the behavior of the system as a Markov chain and provides the outage probability of the system in a closed form using the precoding decision as a parameter; the minimization of the outage expression provides the optimal precoding decision. It turns out that this decision is binary, i.e., throughout its operation, the relay chooses one precoding scheme only. As a result, instantaneous knowledge of all the channel conditions is not required, thus reducing also the communication overhead. The proposed scheme is useful for network scenarios with very critical energy/complexity/bandwidth constraints such as sensor and ad-hoc networks.

In Section II, the network model assumed in this work is described. In Section III, the precoding matrix is designed such that interference cancellation is achieved either at the receiving antenna of the relay or at the destination nodes. Next, in Section IV the network is modeled as a Markov Chain and an outage performance analysis is presented illustrating the steady-state behavior of the network, based on
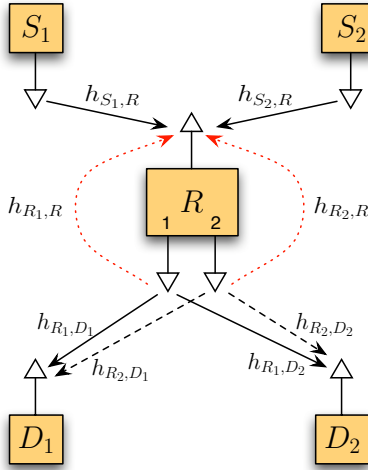
Fig. 1. A simple X-relay configuration consisting of two sources S_1, S_2, two destinations D_1, D_2, and a shared relay R.

the channel conditions. Numerical examples demonstrating the performance of the proposed precoding matrices are given in Section V. Finally, in Section VI conclusions and future directions are drawn.

Notation: We use boldface lower case letters to denote vectors and boldface capital letters to denote matrices. Further, $(\cdot)^{-1}$ and $(\cdot)^T$ stand for matrix inversion and transposition, respectively, whereas $(\cdot)^H$ denotes the Hermitian transposition. $\mathrm{diag}(\cdot)$ returns the diagonal of a matrix, $\mathrm{trace}(\cdot)$ returns the trace of a matrix and $\mathbf{I}$ denotes the identity matrix (with appropriate dimensions). The set of complex numbers is denoted by $\mathbb{C}$. Finally, $\mathbb{P}(X)$ denotes the probability of event X.

II. NETWORK MODEL

We assume a simple X-relay configuration consisting of two sources S_1, S_2, two destinations D_1, D_2, and a shared relay R. The system model considered is depicted in Figure 1. This system model can be regarded as an example of relay-assisted device-to-device (D2D) communications where the source and destination are low-cost devices with some limitations such as a single antenna.

Both sources access simultaneously the channel and each source transmits a message with a spectral efficiency r_0 bits per channel use (BPCU) to the corresponding destination ($S_i \rightarrow D_i$ for $i = 1, 2$). The sources are supposed to always have data traffic to transmit. In order to focus on the relaying process, we assume that a direct link from the sources to the destinations is not available (i.e., direct link does not exist due to large path loss and deep shadowing[1]) and thus communication can be established only via the relay node R. The relay node operates in a full-duplex mode and therefore, it can transmit and receive at the same time and frequency. The nodes S_i, D_i ($i = 1, 2$) are equipped with a single antenna

while the relay node has three antennas in order to implement the full-duplex operation; one antenna for reception and two antennas for transmission[2]. Note that this restriction on the number of antennas demonstrates the problem of not having enough antennas to mitigate interference at both the relay receiving antenna and at the destinations. The channel allows probabilistic receptions of simultaneously transmitted packets (multi-packet reception channel). This is a more realistic and general form of a packet erasure model which captures the effect of fading, attenuation and interference at the physical layer, along with the capability of multi-user detectors at the receiver [13]–[15]. Note that we define outage to be strict in the sense that we consider the case for which either both signals are decoded at the relay or we are in outage. In the case for which only a single signal is decoded at the relay, then the relay has two antennas to multicast a single signal to the destination and there is no MUI at the destinations.

The relay node employs a Decode-and-Forward (DF) strategy and without loss of generality we assume that the processing delay (delay between reception and transmission) is equal to one signal. This means that when the sources transmit the n-th signal, the relay either transmits the $(n-1)$-th sources' signals (given that the decoding of the $(n-1)$-th source signal was successful), or, it remains silent if the decoding was unsuccessful.

Due to the operation of the relay node, a LI is generated at the relay input and a MUI is generated at the destination nodes. These interferences affect the decoding performance of the system. In order to handle interference, the relay node applies a linear precoding matrix $\boldsymbol{T} \in \mathbb{C}^{2 \times 2}$ given by

$$\boldsymbol{T} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \tag{1}$$

on the transmitted signals and therefore each relay antenna transmits a linear combination of the sources' messages. Each node transmits with a fixed power P and thus power control issues are not taken into account. The relay node transmits with a maximum power P. Note that each antenna of the relay transmits with a power $\mathbb{E}(x_{R_i} x_{R_i}^H) = (t_{i1}^2 + t_{i2}^2)P$ and $\mathrm{trace}(\boldsymbol{T}\boldsymbol{T}^H) \leq 1$.

All wireless links exhibit fading and Additive White Gaussian Noise (AWGN) with zero mean and unit variance. The fading is assumed to be frequency non-selective Rayleigh block fading. This means that the fading coefficients $h_{i,j}$ (for the $i \rightarrow j$ link) remain constant during one slot, but change independently from one slot to another according to a circularly symmetric complex Gaussian distribution with zero mean and unit variance.

A global channel state information (CSI) is assumed at the relay node. However, as stated in the introduction, the relay node may not be able to obtain all the channel information (due to hardware and/or complexity constraints) and hence be restricted in partial instantaneous channel state information.

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[1]The model presented in this paper, does not conform to the conventional X-channel setting. In addition, the source-destination direct links are not considered, which makes it closer to a concatenation of a medium access and a broadcast channel.

[2]This configuration is assumed for the sake of exposition and can be generalized to more complex configurations with M antennas for reception and $M + 1$ antennas for transmission.

The received signal at the relay for the n-th time slot, $y_R[n]$, can be expressed as

$$
\begin{aligned}
y_R[n] &= h_{S_1,R}x_1[n] + h_{S_2,R}x_2[n] \\
&\quad + \epsilon[n]\left(h_{R_1,R}x_{R_1}[n] + h_{R_2,R}x_{R_2}[n]\right) + w_R[n], \\
&= h_{S_1,R}x_1[n] + h_{S_2,R}x_2[n] + w_R[n] \\
&\quad + \epsilon[n]h_{R_1,R}\left(t_{11}x_1[n-1] + t_{12}x_2[n-1]\right) \\
&\quad + \epsilon[n]h_{R_2,R}\left(t_{21}x_1[n-1] + t_{22}x_2[n-1]\right), \quad (2)
\end{aligned}
$$

where $x_i[n]$ denotes the n-th signal of the i-th source, $i = 1, 2$, $x_{R_j}[n]$ denotes the n-th signal of the j-th antenna of the relay, $j = 1, 2$, $w[n]$ denotes the AWGN for the n-th time slot, and the indicating factor $\epsilon[n] \in \{0, 1\}$ which indicates if the decoding of the codewords $x_1[n-1]$ and $x_2[n-1]$ at the relay node was successful. More specifically,

$$
\epsilon[n] = \begin{cases} 1, & \text{if } x_1[n-1] \text{ and } x_2[n-1] \text{ decoded successfully;} \\ 0, & \text{otherwise.} \end{cases}
$$

If the decoding of the codewords $x_1[n-1]$ and $x_2[n-1]$ at the relay node is successful, the relay antennas will be active during the transmission of the signals $x_1[n], x_2[n]$; otherwise, if $\epsilon[n-1] = 0$ it means that the decoding at the relay node failed and thus, the relay antennas remain silent at the next slot. This phase of the protocol consists of a conventional multiple-access channel (MAC) with potential interference. On the other hand, the signal received at the i-th destination for the n-th time slot concern the $[n-1]$-th sources' signals and can be written as

$$
\begin{aligned}
y_{D_i}[n] &= \epsilon[n]\left(h_{R_1,D_i}x_{R_1}[n] + h_{R_2,D_i}x_{R_2}[n]\right) + w_{D_i}[n], \\
&= \epsilon[n]\bigg(h_{R_1,D_i}\left(t_{11}x_1[n-1] + t_{12}x_2[n-1]\right) \\
&\quad + h_{R_2,D_i}\left(t_{21}x_1[n-1] + t_{22}x_2[n-1]\right)\bigg) + w_{D_i}[n], \\
&\quad\quad (3)
\end{aligned}
$$

where $w_{D_i}[n]$ denotes the AWGN at the D_i destination.

III. PRECODING DESIGN FOR INTERFERENCE CANCELLATION

For the considered full-duplex X-relay channel, there are two different types of interference that degrade system's performance. More specifically, we have (a) a loop interference that affects the relay input and thus the MAC decoding performance at the relay node, and (b) a multi-user interference that affects the single-user decoding performance at each destination (for the case where the relay transmit data towards the two destinations). The relay node can use the precoding matrix in order to remove either the loop interference or the multi-user interference. The decision aims to minimize the outage probability of the system and it follows the rule given in Theorem 1, which provides easily verifiable conditions that do not require complex calculations and full knowledge of the channel states.

Remark 1. *Note that the relay can simply subtract the loop interference from the received signal and decode the received signal loop interference free. As aforementioned, time-domain cancellation suffers from residual LI that scales with the transmitted relays's power and significantly affects the system's performance by resulting in a zero-diversity order. Under the assumption that any line-of-sight component is efficiently reduced by antenna isolation and the major effects comes from scattering, the LI channel is modeled via the Rayleigh fading distribution (which is a common assumption made in the literature [2]). Hence, given that h_{R_iR} denotes the instantaneous residual LI between a transmitting antenna R_i, $i = 1, 2$ and the receiving antenna R, it follows a complex Gaussian distribution, i.e., $h_{R_iR} \sim \mathcal{CN}(0, \sigma^2_{R_iR})$, where $\sigma^2_{R_iR}$ depends on the distance between the transmit antenna R_i and the receive antenna R of the relay, as well as the capability of the hardware loop interference cancellation (LIC) technique [16].*

For the two cases (LI and MUI), the precoding matrix is defined as follows.

A. Loop Interference Cancellation (LIC)

The goal is to design the precoding matrix $\boldsymbol{T} \in \mathbb{C}^{2\times 2}$ such that, at each time slot n the two signals ($x_{R_1}[n]$ and $x_{R_2}[n]$) transmitted from the relay are eliminated completely. This suggests that the inner product $(h_{R_1,R} \ h_{R_2,R})(t_{1i} \ t_{2i})^T = 0$ for $i = 1, 2$. As a result, $\boldsymbol{T}$ is chosen such that the following equation holds:

$$
\begin{pmatrix} h_{R_1,R} & h_{R_2,R} \end{pmatrix} \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} = \begin{pmatrix} 0 & 0 \end{pmatrix}. \quad (4)
$$

As a result, our problem reduces to specifying t_{21} and t_{22}, such that condition (4) is satisfied. We let $t_{21} = \alpha_1$ and $t_{22} = \alpha_2$, where $\alpha_1, \alpha_2 \in \mathbb{C}$. Although any α_1, α_2 satisfying the above conditions force the LI to zero, in the work we propose a LIC precoding that simultaneously optimizes the decoding performance at each destination, i.e., α_1 and α_2 can possibly be chosen in such a way in order to minimize the outage probability at the destinations.

B. Multi-User Interference Cancellation (MUIC)

Similarly, the goal in this case is to design the precoding matrix $\boldsymbol{T} \in \mathbb{C}^{2\times 2}$ such that, at each time slot n signals $x_1[n]$ and $x_2[n]$ are eliminated completely at destinations D_2 and D_1, respectively. This suggests that the following equation should hold:

$$
\begin{pmatrix} h_{R_1,D_1} & h_{R_2,D_1} \\ h_{R_1,D_2} & h_{R_2,D_2} \end{pmatrix} \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix} \quad (5)
$$

where β_i, $i = \{1, 2\}$ are constants, the values of which depend on the channel states and the precoding matrix chosen. For this case, an efficient solution that has been proposed in the literature is the ZF precoder where $\boldsymbol{T} = \boldsymbol{H}^H(\boldsymbol{HH}^H)^{-1}$ with (see also the seminal work in [17])

$$
\boldsymbol{H} = \begin{pmatrix} h_{R_1,D_1} & h_{R_2,D_1} \\ h_{R_1,D_2} & h_{R_2,D_2} \end{pmatrix}. \quad (6)
$$

Without loss of generality we consider the transmission of the data flow $x_1[n]$. We define the following events:

$$A_1 \triangleq \{\text{R decodes } x_1[n], \, x_2[n] \text{ with LI}\}$$

$$A_2 \triangleq \{\text{R decodes } x_1[n], \, x_2[n] \text{ without LI}\}$$

$$B_1 \triangleq \{D_1 \text{ decodes } x_1[n] \text{ with MUI}\}$$

$$B_2 \triangleq \{D_1 \text{ decodes } x_1[n] \text{ without MUI}\}$$

$$V_1 \triangleq \{\text{precoding matrix } \boldsymbol{T} \text{ suppresses LI}\}$$

$$V_2 \triangleq \{\text{precoding matrix } \boldsymbol{T} \text{ suppresses MUI}\}$$

$$Y \triangleq \{\text{R decodes } x_1[n-1], \, x_2[n-1]\}$$

$$B \triangleq \{D_1 \text{ decodes } x_1[n-1]\}$$

Let v denote the probability of suppressing multi-user interference via an appropriate precoding matrix $\boldsymbol{T}$; i.e. $\mathbb{P}(V_2) = v$, $v \in [0,1]$; thus, $\mathbb{P}(V_1) = 1-v$. Note that the outage probability of decoding a signal from a source at the relay when there is no interference is given by $1 - \mathbb{P}(A_2)$.

A. Markov Chain Representation

At every time instant the network as a system, depending on the events occurring, can be at one of the following four states: $S_1 = \{00\}$ in which neither the relay nor the destinations have decoded a signal; $S_2 = \{01\}$ in which the relay did not decode the received signal but the destinations did decode the signal; $S_3 = \{10\}$ in which the relay only decodes a signal, and $S_4 = \{11\}$ in which both the relay and the destinations decode the received signals. If a signal is not decoded at a destination it is lost and hence has no effect on the network. If a signal is lost at the relay, then the relay does not transmit anything to the destinations and hence there is no loop interference.

The network as a system can be modeled as a Markov chain with the states as just aforementioned. The transition probabilities from and towards these states are summarised below. Note that $\overline{X}$ denotes that event X did not occur.

$$\mathbb{P}(Y \cap B|Y) = \mathbb{P}(V_1)\,\mathbb{P}(A_2 \cap B_1) + \mathbb{P}(V_2)\,\mathbb{P}(A_1 \cap B_2)$$

$$\mathbb{P}(Y \cap B|\overline{Y}) = 0$$

$$\mathbb{P}(Y \cap \overline{B}|Y) = \mathbb{P}(V_1)\,\mathbb{P}(A_2 \cap \overline{B_1}) + \mathbb{P}(V_2)\,\mathbb{P}(A_1 \cap \overline{B_2})$$

$$\mathbb{P}(Y \cap \overline{B}|\overline{Y}) = \mathbb{P}(A_2)$$

$$\mathbb{P}(\overline{Y} \cap B|Y) = \mathbb{P}(V_1)\,\mathbb{P}(\overline{A_2} \cap B_1) + \mathbb{P}(V_2)\,\mathbb{P}(\overline{A_1} \cap B_2)$$

$$\mathbb{P}(\overline{Y} \cap B|\overline{Y}) = 0$$

$$\mathbb{P}(\overline{Y} \cap \overline{B}|Y) = \mathbb{P}(V_1)\,\mathbb{P}(\overline{A_2} \cap \overline{B_1}) + \mathbb{P}(V_2)\,\mathbb{P}(\overline{A_1} \cap \overline{B_2})$$

$$\mathbb{P}(\overline{Y} \cap \overline{B}|\overline{Y}) = 1 - \mathbb{P}(A_2)$$

For simplicity of notation and ease of exposition, we let $\mathbb{P}(Y \cap B|Y) \triangleq p_1$, $\mathbb{P}(Y \cap \overline{B}|Y) \triangleq p_2$, $\mathbb{P}(\overline{Y} \cap B|Y) \triangleq p_3$ and $\mathbb{P}(\overline{Y} \cap \overline{B}|Y) \triangleq p_4$. Also, we let the individual probabilities $p_{X_i} \triangleq \mathbb{P}(X_i)$, where $X \in \{A, B\}$ and $i \in \{1, 2\}$. As a result,

the Markov chain can be characterized by the matrix

$$\mathbf{M} = \begin{pmatrix} 1 - p_{A_2} & 1 - p_{A_2} & p_4 & p_4 \\ 0 & 0 & p_3 & p_3 \\ p_{A_2} & p_{A_2} & p_2 & p_2 \\ 0 & 0 & p_1 & p_1 \end{pmatrix} \tag{7}$$

The Markov chain $\mathbf{M}$ is depicted in Figure 2.

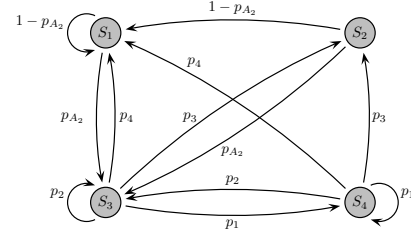
Fig. 2. The network can be modelled as a Markov chain in which there exist 4 states ($\{S_1, S_2, S_3, S_4\} = \{00, 01, 10, 11\}$) comprising the combinations of successful and unsuccessful decoding at the relay and the nodes.

Remark 2. *The probabilities p_{A_1}, p_{A_2}, p_{B_1} and p_{B_2} can be estimated by means of some average information on error detection (via an error detection scheme, for instance, the cyclic redundancy check) of the transmitted packets. The estimation error of such schemes, however, is out of the scope of this work.*

Proposition 1. *The Markov chain $\mathbf{M}$ is Stochastic, Indecomposable and Aperiodic (SIA).*

Proof: (i) By construction $\mathbf{M}$ is a column stochastic matrix. (ii) Since all links receive nonnegative weights and the graph is strongly connected it is implied the matrix $\mathbf{M}$ is indecomposable. (iii) Aperiodicity of the graph is established due to the fact that at least one of the diagonal entries are nonzero. $\blacksquare$

This property establishes that the Markov chain has stationary distribution $\boldsymbol{\pi} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \end{pmatrix}^T$, i.e., $\mathbf{M}\boldsymbol{\pi} = \boldsymbol{\pi}$. Now, we express probabilities p_1, p_2, p_3 and p_4 in terms of v, p_{A_1}, p_{A_2}, p_{B_1} and p_{B_2}. Note that p_{A_1} and p_{B_2} are mutually independent; likewise p_{A_2} and p_{B_1}. Therefore,

$$p_1 = (1-v)p_{A_2}p_{B_1} + vp_{A_1}p_{B_2} \tag{8a}$$

$$p_2 = (1-v)p_{A_2}(1-p_{B_1}) + vp_{A_1}(1-p_{B_2}) \tag{8b}$$

$$p_3 = (1-v)(1-p_{A_2})p_{B_1} + v(1-p_{A_1})p_{B_2} \tag{8c}$$

$$p_4 = 1 - (p_1 + p_2 + p_3) \tag{8d}$$

Then, given the probabilities p_{A_1}, p_{A_2}, p_{B_1} and p_{B_2}, we may be able to choose v such that a desired performance metric is achieved. For example, one may wish to maximize π_1, that is, the probability with which both the relay and the destination nodes will decode the signals received. If we can achieve $\pi_1 = 1$, this is equivalent to $p_{A_2}p_{B_1} = 1$ (and $v = 0$) or $p_{A_1}p_{B_2}$ (and $v = 1$), or both (v can be any value between 0 and 1).

Due to the iterative nature of the protocol, the decoding of each signal at the relay depends on the decoding of the

$$E = \left(\overline{Y} \bigcap \overline{A}_2 \right) \bigcup Y \bigcap \left[\left[V_1 \bigcap \left(\overline{A}_2 \bigcup \left(A_2 \bigcap \overline{B}_1 \right) \right) \right] \bigcup \left[V_2 \bigcap \left(\overline{A}_1 \bigcup \left(A_1 \bigcap \overline{B}_2 \right) \right) \right] \right]. \tag{9}$$

$$\mathbb{P}(E) = 1 - p_{A_2} + p_{A_2} \frac{p_{A_2}(1 - p_{B_1}) + v(p_{A_2}p_{B_1} - p_{A_1}p_{B_2})}{1 + v\left(p_{A_2} - p_{A_1}\right)}. \tag{10}$$

previous signal. More specifically,

$$\mathbb{P}(Y)[n+1] = \left(1 - \mathbb{P}(Y)[n] \right) p_{A_2} +$$
$$\mathbb{P}(Y)[n] \left(p_{A_1} \mathbb{P}(V_2) + p_{A_2} \mathbb{P}(V_1) \right).$$

Since the Markov chain is SIA, it has a steady state [18]. Therefore, by substituting $\mathbb{P}(V_1) = 1 - v$ and $\mathbb{P}(V_2) = v$, at steady state the probability of event Y is given by

$$\mathbb{P}(Y) = \left(1 - \mathbb{P}(Y) \right) p_{A_2} + \mathbb{P}(Y) \left(p_{A_1} v + p_{A_2}(1 - v) \right).$$

Hence, by substituting $\mathbb{P}(V_1) = 1 - v$ and $\mathbb{P}(V_2) = v$ we get

$$p_Y \triangleq \mathbb{P}(Y) = \frac{p_{A_2}}{1 + p_{A_2} - p_{A_1}v - p_{A_2}(1-v)}$$
$$= \frac{p_{A_2}}{1 + v\left(p_{A_2} - p_{A_1}\right)}. \tag{11}$$

As a result, given the probabilities of the relay decoding $x_1[n]$ and $x_2[n]$ with or without LI, p_{A_1} and p_{A_2} respectively, we are able to find the percentage of the decoded signals $x_1[n]$ and $x_2[n]$, or otherwise, the probability with which the signals $x_1[n]$ and $x_2[n]$ at any given time are decoded successfully. Note that in order to maximize $\mathbb{P}(Y)$, Equation (11) suggests that $v = 0$, since $p_{A_2} \geq p_{A_1}$ (equality holds when $p_{A_2} = p_{A_1} = 1$). This is expected, since the loop interference should be cancelled in order to maximize the probability of decoding a packet at the relay node.

B. Optimal precoding decision

In this subsection, we show how a relay should decide optimally on which precoding scheme to choose between LIC and MUIC, such that the outage probability of the system is minimized.

Theorem 1. *Given the probabilities p_{A_1} and p_{A_2} of successful decoding at the relay with or without loop interference respectively, and the probabilities p_{B_1} and p_{B_2} of successful decoding at the destinations with or without multi-user interference respectively, the outage probability of the system $\mathbb{P}(E)$ is minimized if for*
1. $p_{A_2}p_{B_1} - p_{A_1}p_{B_2} > p_{A_2}(1-p_{B_1})(p_{A_2}-p_{A_1})$ the precoding matrix suppresses the loop interference;
2. $p_{A_2}p_{B_1} - p_{A_1}p_{B_2} < p_{A_2}(1-p_{B_1})(p_{A_2}-p_{A_1})$ the precoding matrix suppresses the multi-user interference.
When $p_{A_2}p_{B_1} - p_{A_1}p_{B_2} = p_{A_2}(1 - p_{B_1})(p_{A_2} - p_{A_1})$ any choice of precoding matrix yields the same results.

Theorem 1 provides easily verifiable conditions that do not require complex calculations and full knowledge of the

channel states in order to determine which precoding scheme to choose so that the outage probability of the system is minimized. Once the precoding scheme is chosen, then the outage probability $\mathbb{P}(E)$ is calculated via Equation (10).

Proof: An error event can be expressed as in Equation (9). In steady state, the outage probability $\mathbb{P}(E)$ can be expressed as in Equation (10), by using equation (11). Now, if we want to minimize the outage probability of the system it is enough to find the solution to the following minimization problem,

$$\min_{v \in [0,1]} \frac{p_{A_2}(1 - p_{B_1}) + v(p_{A_2}p_{B_1} - p_{A_1}p_{B_2})}{1 + v\left(p_{A_2} - p_{A_1}\right)}. \tag{12}$$

The minimization problem (12) is equivalent to

$$\min_{v \in [0,1]} \frac{a + bv}{1 + vc}, \tag{13}$$

where $a \triangleq p_{A_2}(1 - p_{B_1})$, $b \triangleq p_{A_2}p_{B_1} - p_{A_1}p_{B_2}$ and $c \triangleq p_{A_2} - p_{A_1}$. In general, $p_{A_2} \geq p_{A_1}$ and hence $c \geq 0$. If $b \leq 0$, then a natural choice that minimizes $\mathbb{P}(E)$ is $v = 1$. However, if $b > 0$, then the optimization problem (12) should be solved in order to find the optimal strategy. Towards this end, by defining

$$J(v) \triangleq \frac{a + bv}{1 + vc}, \tag{14}$$

and by taking the derivative of $J(v)$ with respect to v, we obtain

$$\frac{dJ(v)}{dv} = \frac{b - ac}{(1 + vc)^2}. \tag{15}$$

Therefore, if $b > (<)ac$, then $J(v)$ is increasing (decreasing) with v and hence it is minimized by taking $v = 0$ ($v = 1$). If $b = ac$, then $J(v) = a$ and any $v \in [0,1]$ would work. ∎

V. EXAMPLES

A. Illustrative numerical example

Let, for example, $p_{A_1} = 0.6$, $p_{A_2} = 0.7$, $p_{B_1} = 0.4$ and $p_{B_2} = 0.5$. Then, we are able to check our condition:

$$p_{A_2}p_{B_1} - p_{A_1}p_{B_2} = 0.7 \times 0.4 - 0.6 \times 0.5 = -0.02$$
$$p_{A_2}(1 - p_{B_1})(p_{A_2} - p_{A_1}) = 0.7(1 - 0.4)(0.7 - 0.6) > 0$$
$$> p_{A_2}p_{B_1} - p_{A_1}p_{B_2}.$$

Hence, according to Theorem 1, the precoding matrix suppresses the MUI and therefore, $v = 1$. Having found v, now we are able to find via Equation (10) that the outage probability is $\mathbb{P}(E) = 0.5545$, while the probability of decoding $x_1[n]$ and $x_2[n]$ at the relay node is $\mathbb{P}(Y) = 0.6364$. If we erroneously have chosen to suppress LI, then we would obviously achieve better probability of decoding $x_1[n]$ and $x_2[n]$ at the relay node

$(\mathbb{P}(Y) = 0.6364)$, but the outage probability of the system would be worse ($\mathbb{P}(E) = 0.5940$).

B. Simulations

We next provide the following scenario: the probability of successful decoding at the relay without LI and at the destination without MUI is set to 1, i.e., $p_{A_2} = p_{B_2} = 1$. Then, we let $p(A_1)$ and $p(B_1)$ vary and we compare the outage probabilities of the optimal precoding scheme with the erroneous. In Figure 3, the optimal (erroneous) precoding scheme appears in light blue (light brown).
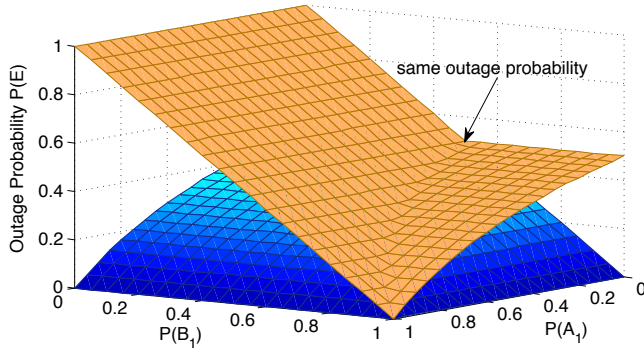

Fig. 3. Outage probability $\mathbb{P}(E)$ with optimal precoding scheme (light blue) and erroneous precoding scheme (light brown). There is only a line for which the optimal and the erroneous precoding give the same outage probability; this occurs when $p_{A_2}p_{B_1} - p_{A_1}p_{B_2} = p_{A_2}(1 - p_{B_1})(p_{A_2} - p_{A_1})$.

There is only a single line for which the optimal and the erroneous precoding give the same outage probability. This occurs when $p_{A_2}p_{B_1} - p_{A_1}p_{B_2} = p_{A_2}(1-p_{B_1})(p_{A_2} - p_{A_1})$, as stated in Theorem 1. Otherwise, the gain obtained by optimal precoding scheme can be tremendous, as shown in Figure 3.

VI. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper, a simple X-relay configuration was studied in which the shared relay operates in FD mode. We investigated the case in which the relay node may not be able to handle both the LI and the MUI due to critical energy/complexity/bandwidth constraints. As a result, a precoding decision is made based only on statistical knowledge of the channel conditions. The system was formulated as a Markov chain and the outage probability was derived in closed-form with the precoding decision as a parameter. It turned out that the outage probability is minimized with a binary precoding decision and simple verifiable conditions were proposed for deciding on the precoding decision.

This work reveals many open issues that are part of ongoing or future work. More specifically,

- it is important to study the achievable rates of this setup, which is essentially limited by the bottleneck link;
- it would be also interesting to investigate the case in which the relay can dynamically choose to cancel either the LI or the MUI; and,

- it is interesting to consider the case when the relay is buffer-aided (see, e.g., [19]–[22]) and an opportunistic transmission (buffering or forwarding) can be adopted.

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