Reduced power expenditure in the minimum latency transmission scheduling problem

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***Abstract*—In large networks due to the large number of transmissions it is unlikely that a slot will be used only by a single wireless node. In this paper, a Cutting Plane approach is proposed in order to combine the minimum latency transmission scheduling problem with power expenditure minimization. The problem is partitioned into a Master Problem and a Subproblem; the main objective of the Subproblem is to minimize the total power expenditure. The Cutting Plane algorithm used is evaluated with regard to its performance and the outcome is that the power expenditure is reduced significantly, while achieving the minimum latency transmission scheduling. One of the drawbacks of the proposed approach, however, is the fact that although it reduces the power significantly, the computational time required is quite large, pointing to further research towards that direction.**

***Index Terms*—transmission scheduling, SINR, power control, minimum latency.**

I. Introduction

In large networks there exist inevitably many transmissions and it is therefore unlikely that a slot (time-slot or channel) will be used only by a single wireless node. Therefore, methods that provide bandwidth guarantees to wireless connections as well as optimize transmission scheduling are essential. The design of multiple access mechanisms to efficiently control channel access is of vital importance.

The power of each transmitter in a wireless network is directly related to the resource usage of the link and it is a valuable resource, since the batteries of the wireless nodes have limited lifetime. Power control has been extensively employed for Medium Access Control (MAC) in multi-hop wireless networks (for example in [1]–[6]). Some of them aim to minimize power dissipation. For example, [7] proposes a two-phase method for the joint scheduling and power control which aims to find an admissible set of links along with their transmission power levels in a single channel only. On the other hand, the minimum latency transmission scheduling problem has been thoroughly investigated (see, for example, [8] and references therein).

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This work considers multiple channels and it solves the minimum latency transmission scheduling problem under the physical model. The goal is to assign channels (or time slots) and transmitting powers to communication links such that all communication requests are processed correctly, specified Quality-of-Service (QoS) requirements are met, and the number of required slots (time-slots or channels) is minimized. This problem has been studied extensively in the recent past [8]–[11], but this is the first time, to the best of the authors' knowledge, that an attempt is being made to reduce the power expenditure on top of minimizing the number of slots required for scheduling transmission. More specifically, we partition the problem into a Master problem and a Subproblem. While the main objective of the Master problem is to minimize the latency of the network, the objective of the subproblem is to minimize the total power expenditure in the network. This leads to a considerable reduction of the power expenditure with respect to other approaches (as it is shown in our comparisons), without guaranteeing that the final realization is also the optimal. While this approach offers reduced power expenditure, the computational time required is still quite large, suggesting that further research is needed towards that direction.

The remainder of the paper is organized as follows. In Section II the model under consideration along with the basic terminology and formulas is introduced. In Section III the problem is formulated as a Mixed Integer Program (MIP). The MIP is then partitioned and solved through a Cutting Plane (CP) approach in Section IV. In Section V, the performance of the CP algorithm is evaluated. Concluding remarks and future directions are given in Section VI.

II. Model and Preliminaries

In this study, we consider a network where the links are assumed to be unidirectional and each node is supported by an omnidirectional antenna. Each node can only be a receiver or a transmitter at each time instant due to the half-duplex nature of the wireless transceiver. Each transmitter aims to communicate with a single node (receiver) only, which cannot receive from more than one node simultaneously. We denote by $\mathcal{T}$ the set of transmitters and $\mathcal{R}$ the set of receivers in the network.

The link quality is measured by the Signal-to-Interference-and-Noise-Ratio (SINR). The channel gain on the link between transmitter i and receiver j is denoted by g_{ij} and incorporates the mean path-loss as a function of distance, shadowing and fading, as well as cross-correlations between signature sequences. All the g_{ij}'s are positive and can take values in the range $(0, 1]$. Without loss of generality, we assume that the intended receiver of transmitter i is also indexed by i. The power level chosen by transmitter i is denoted by p_i. ν_i denotes the variance of thermal noise at the receiver i, which is assumed to be additive Gaussian noise.

The interference power at the i^{th} node, I_i, includes the interference from all the transmitters in the network and the thermal noise, and is given by

$$I_i(\mathbf{p}) = \sum_{j \neq i, j \in \mathcal{T}} g_{ji} p_j + \nu_i. \tag{1}$$

where $\mathbf{p} = [p_1 \;\; p_2 \;\; \ldots \;\; p_{|\mathcal{T}|}]^T$. Therefore, the SINR at the receiver i is given by

$$\Gamma_i(\mathbf{p}) = \frac{g_{ii} p_i}{\sum_{j \neq i, j \in \mathcal{T}} g_{ji} p_j + \nu_i} \;. \tag{2}$$

Due to the unreliability of the wireless links, it is necessary to ensure Quality of Service (QoS) in terms of SINR in wireless networks. Hence, independently of nodal distribution and traffic pattern, a transmission from transmitter i to its corresponding receiver is successful (error-free) if the SINR of the receiver is greater or equal to the *capture ratio* γ_i ($\Gamma_i \geq \gamma_i$). The value of γ_i depends on the modulation and coding characteristics of the radio. Therefore,

$$\frac{g_{ii} p_i}{\sum_{j \neq i, j \in \mathcal{T}} g_{ji} p_j + \nu_i} \geq \gamma_i \tag{3}$$

Inequality (3) depicts the QoS requirement of a communication pair i while transmission takes place. After manipulation it becomes equivalent to the following

$$p_i \geq \gamma_i \left(\sum_{j \neq i, j \in \mathcal{T}} \frac{g_{ji}}{g_{ii}} p_j + \frac{\nu_i}{g_{ii}} \right). \tag{4}$$

III. Problem Formulation

In this section, we present the problem of finding the minimum number of slots (time slots or channels) and the corresponding transmitting powers, such that all communication requests are being processed correctly and Quality of Service (QoS) requirements for successful transmissions are satisfied. Note that to ensure feasibility of our problem we define the maximum number of slots that may be required to be equal to the number of links, $|\mathcal{L}|$. Henceforth, we will assume that the first slot is at time 1; the latest point in time for which there can be a scheduled transmission is therefore $D = |\mathcal{L}|$.

To formulate the optimization problem, we define two sets of decision variables, for each transmitter $i \in \mathcal{T}$ and time $t = 1, \ldots, D$; processing-time variables:

$$x_i(t) = \begin{cases} 1, & \text{if transmitter } i \text{ is active at time } t \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

and power level variables: $p_i(t) \in \mathbb{R}_+$.

Since the problem involves both integer and continuous decision variables, the mathematical formulation is classified as a Mixed Integer Program (MIP) and is given in Model 1.

Model 1 Minimum number of time slots

$$\underset{t,x,p}{\textbf{minimize}} \;\; \tau = \max_{i \in \mathcal{T}} \sum_{t=1}^{D} t x_i(t) \tag{6a}$$

subject to

$$\sum_{t=1}^{D} x_i(t) \geq 1 \; \forall\, i \in \mathcal{T}, \tag{6b}$$

$$x_i(t) = 0 \Rightarrow p_i(t) = 0 \; \forall\, i \in \mathcal{T}, \; t = 1, \ldots, D, \tag{6c}$$

$$x_i(t) = 1 \Rightarrow g_{ii} p_i(t) \geq \gamma_i \left(\sum_{j \in \mathcal{T}, j \neq i} g_{ji} p_j(t) + \nu_i \right) \tag{6d}$$

$$\forall\, i \in \mathcal{T}, \; t = 1, \ldots, D,$$

$$x_i(t) \in \{0, 1\} \;\; \forall\, i \in \mathcal{T}, \; t = 1, \ldots, D, \tag{6e}$$

$$p_i(t) \in \mathbb{R}_+ \; \forall\, i \in \mathcal{T}, \; t = 1, \ldots, D. \tag{6f}$$

Objective (6a) minimizes the number of time slots needed to schedule all the transmitters in the network. τ is equal to the latest point in time needed for scheduling a transmission. Constraint (6b) ensures that each link in the network is processed at least once in the schedule. Note that there is always an optimal schedule in which each link is processed only once. Constraint (6c) makes sure that if a pair is not processed at a specific time slot, then the power level of the corresponding transmitter is 0 at that time slot. The QoS conditions are guaranteed by constraint (6d). The constraint only affects the optimization if $x_i(t)$ takes the value 1. Finally, the last two constraints (6e) and (6f) define the admissible values for the decision variables.

IV. A Cutting Plane Approach

We propose a Cutting Plane approach, referred to as CP_{slots}, to construct the "Minimum latency transmission scheduling with SINR constraints" problem (Model 1). The basic idea is to partition the problem into a *Master Problem* and a *Subproblem*. The iterative procedure solves the Master Problem first and uses the obtained optimal decision values to solve the Subproblem. Depending on whether the Subproblem is feasible or infeasible, *cuts* may be derived. These cuts are then added to the Master Problem and the procedure is repeated. The algorithm terminates when the solution obtained at any iteration satisfies the optimality/termination criteria.

Problem partitioning. Model 1 may be partitioned into a Master Problem (Model 2) and a Subproblem (Model 3), as shown next:

Model 2 Master problem

$$\underset{x}{\textbf{minimize}} \;\; \tau = \max_{i \in \mathcal{T}} \sum_{t=1}^{D} t x_i(t) \tag{7a}$$

subject to

$$\sum_{t=1}^{D} x_i(t) \geqslant 1 \; \forall i \in \mathcal{T} \tag{7b}$$

$$x_i(t) \in \{0, 1\} \; \forall i \in \mathcal{T}, t = 1, \ldots, D. \tag{7c}$$

Model 3 Subproblem

$$\underset{p}{\textbf{minimize}} \sum_{i\in\mathcal{T}}\sum_{t=1}^{D} p_i(t) \tag{8a}$$

subject to

$$x_i^*(t) = 0 \Rightarrow p_i(t) = 0 \;\forall\, i \in \mathcal{T},\; t = 1,\dots,D, \tag{8b}$$

$$x_i^*(t) = 1 \Rightarrow p_i(t) g_{ii} \geq \gamma_i \left(\sum_{j\in\mathcal{T}, j\neq i} g_{ji} p_j(t) + \nu_i \right) \tag{8c}$$

$\forall\, i \in \mathcal{T},\; t = 1,\dots,D,$

$$p_i(t) \geqslant 0 \;\forall i \in \mathcal{T},\, t = 1,\dots,D. \tag{8d}$$

Let $x_i^*(t,k)$ denote the optimal values of the processing time variables obtained by solving Model 2 at iteration k. For brevity, we will denote these as $x^*(k)$. Using $x^*(k)$, Model 3 decomposes into distinct problems for each time t derived from $x^*(k)$. Each of the decomposed problems corresponds to a time t and reduces to the problem of finding the minimum total transmission power for the schedule defined by all transmissions i such that $x_i^*(t,k) = 1$. In fact, the objective function of the Subproblem may be changed as we are only testing the feasibility of the solution derived from the Master problem.

Given $x^*(k)$, the Subproblem may be infeasible; the schedule derived from the Master problem may assign the same slot to transmissions which cannot be simultaneously processed without violating SINR constraints.

Cut generation. If there is a time t such that the simultaneous transmission of all $i \in \mathcal{T}$ for which $x_i^*(t,k) = 1$ is infeasible with respect to the Subproblem (Model 3), the following feasibility cuts may be applied to the Master problem (Model 2). Let $EX_t(k) = \{i \in \mathcal{T} | x_i^*(t,k) = 1\}$. The following cut forbids the simultaneous processing of all the transmission pairs in $EX_t(k)$:

$$\sum_{i\in EX_t(k)} x_i(t') < |EX_t(k)|, \qquad \forall t' = 1,\dots,D \tag{9}$$

These cuts should be applied for all t for which the corresponding scheduled transmissions are violating the SINR constraints. The cuts are valid feasibility cuts for the original problem and, as such, can be imposed at each iteration.

Remark 1. *Additional to the cuts applied at each iteration of the algorithm, we can apply cuts from information known* a priori. *Such cuts show in our simulations that help in achieving a quicker convergence to the optimal solution.*

Termination criteria. Let $MP^*(k)$ denote the value of the optimal solution to Model 2 obtained at iteration k. τ_{min} denotes the value of the optimal solution to Model 1. The Cutting Plane algorithm proceeds iteratively until Model 3 becomes feasible. This termination criterion guarantees the optimality of the solution; we provide a proof for this below.

Proof. 1) At each iteration k, the Master Problem (Model 2) is a relaxation of the original problem (Model 1). Hence, its optimal solution value, $MP^*(k)$, is a lower bound to τ_{min}, i.e. $MP^*(k) \leq \tau_{min}$.

2) At each iteration k, and before the algorithm terminates, a new feasibility cut (distinct from cuts found at all previous iterations) is found by solving the Subproblem.

3) Since there are only a finite number of cuts available (corresponding to each possible set of transmission pairs), the algorithm is guaranteed to converge. For the final iteration K, the Subproblem is feasible. Therefore, the values of the processing time variables for that iteration represent a feasible solution to the original problem. Hence, $MP^*(K) \geq \tau_{min}$. Since $MP^*(K) \leq \tau_{min}$ (from the first point)), we have $MP^*(K) = \tau_{min}$, and thus convergence to the optimal solution has been achieved. □

The complete Cutting Plane algorithm is summarized below.

Algorithm 1 A Cutting plane approach for the "Minimum latency transmission scheduling with SINR constraints" problem

initialise
  $k = 1$
  Compute LB {*Lower bound, e.g., from [8]*}
  Compute UB {*Upper bound, e.g., from [8]* }
  $feas = false$ {*feasibility indicator, obtained from solving the Subproblem (Model 3)*}
   Add cuts to Model 2.
if $LB < UB$ **then**
  repeat
    Solve Model 2 to get optimal values $x^*(k)$ and $MP^*(k)$. Set $LB = \max[MP^*(k), LB]$.
    Solve Model 3 to check feasibility of SINR constraints.
    if Model 3 is feasible **then**
      $feas = true$
    else
      Add cuts of the form (9) to Model 2.
    end if
    $k = k + 1$
  until $feas = true$ or $LB = UB$
end if
$k = k - 1$
return LB, $x^*(K)$.

The algorithm begins with the computation of a lower bound (LB) and upper bound (UB) according to techniques described in [8] (not described here since it is out of the scope of this paper). The initial cuts are also determined and added to the Master Problem (Model 2) at this stage. It is evident that if LB has the same value as UB, the Cutting Plane algorithm is redundant and hence, the heuristic solution would be optimal for the transmission scheduling problem.

If $LB < UB$, the problem scale is reduced by setting the deadline of the problem $D = UB$; the Master Problem (Model 2) is then solved and the lower bound of the algorithm is updated. The Subproblem (Model 3) is then examined and the SINR constraints are tested for feasibility. If infeasibilities occur, cuts of the form (9) are added to Model 2 and the process is repeated. The algorithm terminates when the solution obtained from Model 2 is either feasible with respect to the

SINR constraints or the optimal objective value of the Master problem, $MP^*(k)$, at iteration k equals UB (the solution value found by the heuristic at the initialization stage of Algorithm 1). In the latter case, the optimal objective function value for the transmission scheduling problem is equal to UB and the optimal schedule would be the one found using the priority scheduling policy.

V. Performance Evaluation

The algorithm has been coded in Microsoft Visual Studio 2005 C++ using CPLEX v12.1 and run on an Intel Core 2 computer, with 2.5GHz processor and 3.5GB of RAM. The lower bound used in CP_{slots} is LB described in [8]. Further to CP_{slots}, we also develop another Cutting Plane algorithm, referred to as CP_{cuts}, similar to CP_{slots}; however, the initial bounds are not used. For benchmarking purposes, we also code a B&B approach, referred to as BB_{gen}, which is generic as it only implements CPLEX's default settings (no bounds are incorporated) and BB_{slots} in [8] for comparison purposes.

Table I shows the average computational time (in CPU seconds) required to solve the network instances within the set time limit using both CP_{cuts} and CP_{slots}, emphasizing the importance of using initial bounds.

No. of pairs	CPU time (sec)		♯out of time	
	CP_{cuts}	CP_{slots}	CP_{cuts}	CP_{slots}
10	0.69	0.59	0	0
20	44.70	13.96	0	0
30	1840.18	587.23	3	2
40	N/A	548.20	10	4

TABLE I
Numerical Results for finding the optimal solution to Model 1 using CP_{cuts} and CP_{slots}.

It is evident from Table I that CP_{slots} outperforms CP_{cuts} in all sets of test problems; this is especially evident for the largest-sized instances in the table (40-pairs) where CP_{slots} is capable of solving 60% of the instances while none of them could be solved by CP_{cuts} in the set time limit.

Our CP_{slots} are compared with the Branch & Bound (B&B) techniques used in [8]. While CP_{slots} outperforms the generic B&B (BB_{gen}), its computational performance is not as powerful as the performance of the problem specific B&B technique (BB_{slots} in [8]). However, it is worth mentioning that CP_{slots} provides solutions with a smaller total power, on average, than the one provided by BB_{slots}. This is a natural consequence of the objective chosen (8a) in Model 8. Table V shows the average percentage deviation in the sum of the powers found by the solutions provided by BB_{slots} ($power_{bb}$) as compared to the solution found by CP_{slots} ($power_{cp}$).

No. of pairs	$\% \frac{power_{bb}-power_{cp}}{power_{cp}}$
10	53%
20	3%
30	1646%
40	2409%

TABLE II
Numerical Results of total power required by the solution found by BB_{slots} as compared to the solution found by CP_{slots}.

According to Table V, the solution obtained by BB_{slots} is associated to a total power which is on average *significantly larger* than the one provided by CP_{slots}.

It is worth noting though, that CP_{slots} is able to minimize the total power of the *given* solution found by solving the Master Problem (Model 2), but not provide the minimum total power over all optimal solutions to Model 1. In fact, it is possible that the solution provided by BB_{slots} (even though not superior in terms of the number of time slots) could provide a smaller total power than the solution found by CP_{slots}.

VI. Conclusions

In this paper, a Cutting Plane approach is proposed in order to combine the minimum latency transmission scheduling problem with power expenditure minimization. The idea is to partition the problem into a Master Problem and a Subproblem; the objective of the Subproblem is to minimize the total power expenditure. It is shown in our performance evaluation that the power expenditure is reduced significantly.

While the Cutting Plane approach reduces the power significantly, the computational time required in very large and hence this methodology cannot be used in large networks. Current research focuses on finding bounds that guarantee that the error lies within a maximum distance from the optimum solution.

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