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Transmission Scheduling in Wireless Networks with SINR Constraints

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***Abstract*—Despite all the work being done so far in scheduling for wireless networks, the transmission scheduling problem with specific constraints is not yet fully understood and explored. In this work, we find the minimum number of time-slots (or channels) required in any given network and the corresponding transmitting powers, such that all communication requests are being processed correctly, while fulfilling specific constraints that comply with the Quality of Service (QoS) requirements for successful transmissions. More specifically, in this paper, we formulate the optimization problem as a Mixed Integer Program (MIP) and solve the mathematical formulation by a Branch-and-Bound algorithm. Our computational study shows that our methodology is capable of solving exactly networks of varying size but can also provide a near optimal heuristic solution for the hardest instances. The results can be used as bounds for the study of distributed algorithms that aim for the optimal scheduling and power assignment without information about the whole network. Numerical examples are provided to illustrate the validity of our proposed methodology.**

I. Introduction

A wireless node can usually transmit on a single channel from a number of channels that have a fixed range of frequency spectrum. Each node can choose on which channel to transmit. In large networks, where many communication pairs exist, it is unlikely that a channel will be used by a single communication pair only. Hence, the same channel has to be reused by different transmitters. Each transmission corresponds to a spatiotemporal propagation of radio waves that are received by all nodes in proximity utilizing the same channel at the time, leading to interference, the so-called *co-channel interference*.

The interference at the receiver caused by the signals transmitted by nodes, other than the intended transmitter are considered to be noise. Therefore, apart from all these losses the receiver experiences by the wireless channel, there exists interference in the channel that makes the reception of signals even worse. Therefore, the existence of a link between two nodes mainly depends on the transmission power and the interference by the other nodes in the same channel. By increasing the transmission power, the link quality is improved, but at the same time, the interference on other nodes in the channel increases. In reality, the interference between communication pairs is difficult to predict and to control, which results in the instability of link quality. The quality of the established link is measured by the *Signal-to-Interference-and-Noise-Ratio* (SINR).

While the interference model described is considered as more realistic than other existing models (e.g. the *Protocol Model* [1]) and in the case of dynamic power control, it is difficult to incorporate in the problem formulation in which time slot (or which channel) each node should be allowed to transmit and at which power, so that the QoS requirements are guaranteed while at the same time interference mitigation is accomplished for the rest of the nodes in the network. From hereafter, we will be referring to time-slots only, but the same are valid for channels as well.

Thus, it is of vital importance to orchestrate channel access in order to fully exploit spatial reuse and hence, minimize the number of time-slots required to successfully complete all requests. The value of this result is not limited to a centralized solution of the problem, but it also provides a bound for the evaluation of distributed joint power and scheduling algorithms developed. Some previous attempts for the solution of this problem were focused on heuristics (e.g. [2]), which do not necessarily provide an optimal solution. In [3], the link assignment optimization problem was formulated which assigns at least a time-slot to each link such that the number of time-slots is minimized. However, the power levels' assignment is not considered as it is assumed to be given as input into the optimization.

In this work, we solve the open problem addressed in [4]: Given n communication requests, we assign a color (time-slot) to each request. For all requests sharing the same color we specify the power levels such that each request can be handled correctly. Therefore, not only we determine the minimum number of time-slots required such that all the communication pairs have at least a time-slot to communicate at the required SINR constraints, but also, we find the optimal powers that yield this minimum number of time-slots. In this paper, we have used mathematical programming techniques to formulate the problem of scheduling and assigning power levels to the communication pairs in a wireless network so that QoS requirements are satisfied and the total number of time-slots required is minimized.

The rest of the paper is organized as follows. In Section II the network and communication model is described. In Section

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III, the problem formulation is provided and Section IV discusses the methodology used to solve the problem. Numerical examples are then presented in Section V to illustrate the validity of our results. Finally, in Section VI conclusions are drawn and directions for future work are also given.

II. Network and Communication Model

We consider a planar network where the links are assumed to be unidirectional and each node is supported by an omnidirectional antenna. This can be represented by a graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is the set of all nodes and $\mathcal{L}$ is the set of the active links in the network. Each node can be a receiver or a transmitter only at each time instant due to the half-duplex nature of the wireless transceiver. Each transmitter aims to communicate with a single node (receiver) only, which cannot receive from more than one nodes simultaneously. We denote by $\mathcal{T}$ the set of transmitters and $\mathcal{R}$ the set of receivers in the network.

The channel gain on the link between transmitter i and receiver j is denoted by g_{ij} and incorporates the mean path-loss as a function of distance, shadowing and fading, as well as cross-correlations between signature sequences. All the g_{ij}'s are positive and can take values in the range $(0, 1]$. The power level chosen by transmitter i is denoted by p_i and the intended receiver is also indexed by i. ν denotes the variance of thermal noise at the receiver, which is assumed to be additive Gaussian noise. The link quality is measured by the Signal-to-Interference-and-Noise-Ratio (SINR). The interference power at the i^{th} receiver, I_i, includes the interference from all the transmitters in the network (apart from the communicating transmitter) and the thermal noise, and is given by

$$I_i = \sum_{j \neq i, j \in \mathcal{T}} g_{ji} p_j + \nu. \tag{1}$$

Therefore, the SINR at the receiver i, Γ_i, is given by

$$\Gamma_i = \frac{g_{ii} p_i}{\sum_{j \neq i, j \in \mathcal{T}} g_{ji} p_j + \nu} \; . \tag{2}$$

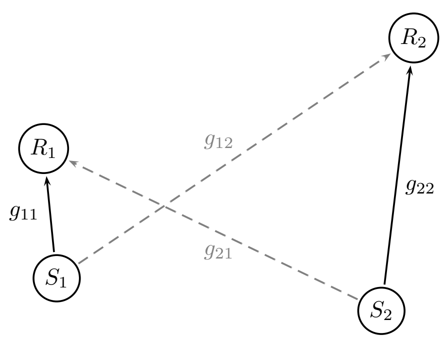


Fig. 1. An example of a network consisting of two communication pairs only. Each pair i consists of a transmitter S_i and a receiver R_i connected with a solid line while the grey dotted arrows indicate the interference that transmitters cause to the neighboring receivers.

Due to the nature of the wireless channel, it is necessary to ensure Quality of Service (QoS) at the wireless links in terms of SINR in wireless networks. Independently of nodal distribution and traffic pattern, a transmission from transmitter i to its corresponding receiver is considered successful if the SINR of the receiver is greater or equal to γ_i ($\Gamma_i \geq \gamma_i$), called the *capture ratio* and is dependent on the modulation and coding characteristics of the radio [5]. Therefore we require,

$$\frac{g_{ii} p_i}{\sum_{j \neq i, j \in \mathcal{T}} g_{ji} p_j + \nu} \geq \gamma_i \tag{3}$$

A. Condition for simultaneous transmission

Equation (3) after manipulation, is equivalent to the following

$$p_i \geq \gamma_i \left(\sum_{j \neq i, j \in \mathcal{T}} \frac{g_{ji}}{g_{ii}} p_j + \frac{\nu}{g_{ii}} \right). \tag{4}$$

In matrix form, for a network consisting of n communication pairs, this can be written as

$$\mathbf{p} \geq \Gamma G \mathbf{p} + \eta \tag{5}$$

where

$$\begin{aligned} &\Gamma = diag(\gamma_i) \\ &\mathbf{p} = \begin{pmatrix} p_1 & p_2 & \dots & p_n \end{pmatrix}^T \\ &G_{ij} = \begin{cases} 0 & \text{, if } i = j, \\ \frac{g_{ji}}{g_{ii}} & \text{, if } i \neq j. \end{cases} \\ &\eta_i = \frac{\gamma_i \nu}{g_{ii}} \end{aligned}$$

Let,

$$C = \Gamma G \tag{6}$$

so that (5) can be written as

$$(I - C)\mathbf{p} \geq \eta \tag{7}$$

The matrix C has nonnegative elements and it is reasonable to assume that is irreducible, since we are not considering totally isolated groups of links that do not interact with each other. By the Perron-Frobenius theorem [6], we have that the spectral radius of the matrix C is a simple eigenvalue, while the corresponding eigenvector is positive componentwise. The necessary and sufficient condition for the existence of a nonnegative solution to inequality (7) for every positive vector η is that $(I - C)^{-1}$ exists and is nonnegative. However, $(I - C)^{-1} \geq 0$ if and only if $\rho(C) < 1$ [7] (*Theorem 2.5.3*), [8] .

Therefore, the necessary and sufficient condition for (7) to have a positive solution $\mathbf{p}^*$ for a positive vector η is that the Perron-Frobenius eigenvalue of the matrix C is less than 1. That is, there exists a set of powers such that all the senders can transmit simultaneously and still meet their QoS requirements (minimum SINR for successful reception).

III. Problem Formulation

The notation used in the paper is given in Table 1:

Notation 1 Notation used in the paper:

$\mathcal{N}$	The set of all nodes in the network
$\mathcal{L}$	The set of active links in the network
$\mathcal{G} = (\mathcal{N}, \mathcal{L})$	The graph of the network
$\mathcal{T}$	The set of transmitters in the network
$\mathcal{R}$	The set of receivers in the network
g_{ij}	The channel gain on the link between transmitter i and receiver j
ν	The variance of thermal noise at the receiver
I_i	The interference power at the i^{th} receiver
Γ_i	The SINR at the i^{th} receiver
γ_i	The capture ratio at the i^{th} receiver
P_i^{max}	The maximum power capacity of the i^{th} receiver
D	The deadline of the network

Note that to ensure feasibility we can define the maximum number of times slots as equal to the number of links, $|\mathcal{L}|$(=$|\mathcal{T}| = |\mathcal{R}|$, in this case). We assume that time starts from $t = 0$, hence the deadline of our problem is set as $D = |\mathcal{L}|-1$.

The problem objective is to select the time-slot for processing each communication pair in the network and to assign power levels to the transmitters so that the total number of time slots needed to process all pairs in the network is minimized whilst satisfying Quality-of-Service requirements. To formulate the optimization problem, we define two sets of decision variables for each communication pair $i \in \mathcal{T}$ and time $t \in [0, D]$:

- Processing-time variables:

$$x_i(t) = \begin{cases} 1, & \text{if transmitter } i \text{ is processed} \\ & \text{at time } t \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

- Power level variables:

$$p_i(t) \in \mathbb{R}^+ \tag{9}$$

Since the problem involves both integer and continuous decision variables, the mathematical formulation is classified as a Mixed Integer Program (MIP) and is given in Model 1.

Model 1 Transmission Scheduling in Wireless Networks with SINR Constraints

Minimize

$$\tau = \max_{i \in \mathcal{T}} \{\sum_{t=0}^{D} t x_i(t)\} \tag{10}$$

subject to:

$$\sum_{t=0}^{D} x_i(t) \geqslant 1 \qquad \forall i \in \mathcal{T} \tag{11}$$

$$p_i(t) \leqslant x_i(t) P_i^{max} \qquad \forall i \in \mathcal{T},\, t \in [0, D] \tag{12}$$

$$p_i(t) g_{ii} \geqslant \gamma_i \{\sum_{j \neq i, j \in \mathcal{T}} g_{ji} p_j(t) + \nu\} - (1 - x_i(t)) M_i \qquad \forall i \in \mathcal{T},\, t \in [0, D] \tag{13}$$

$$x_i(t) \in \{0, 1\} \qquad \forall i \in \mathcal{T},\, t \in [0, D] \tag{14}$$

$$p_i(t) \geqslant 0 \qquad \forall i \in \mathcal{T},\, t \in [0, D] \tag{15}$$

Objective (10) aims to minimize the time slots needed to schedule all the transmitters in the networks. τ is equal to the latest point in time for which there is a communication pair being processed. Constraint (11) ensures that each link in the network is processed at least once in the schedule. Note that in the optimal schedule each link will only be processed once. Constraint (12) makes sure that if a pair is not processed at a specific time slot, then the power level of the corresponding transmitter is 0 at that time slot. Note that the maximum power capacity of the $i^t h$ receiver, P_i^{max}, is considered as given. The QoS conditions are guaranteed by constraint (13). Note that M_i is a number large enough to ensure that if $x_i(t)$ takes the value 0, then the RHS of the constraint is always negative, and therefore the constraint is satisfied. The constraint only affects the optimization if $x_i(t)$ takes the value 1. Finally, the last two constraints (14 and 15) define allowable values for the decision variables.

IV. Solution Approach: Branch-and-Bound

Branch-and-Bound (B&B) is one of the most widely applied and studied implicit enumeration algorithms for finding optimal solutions of various optimization problems involving discrete decision variables. It consists of a systematic construction of candidate solutions and calculations of upper and lower bounds on the objective function to speed up the search process.

The branching process can be seen as building up a tree, where the *relaxed linear* problem under investigation sits in the root and new problems are stored in child nodes. Each child node is a copy of its parent node plus some additional constraints. The B&B algorithm defines which additional constraints are added (*branching strategy*) and which child node is to be solved first (*node selection strategy*).

Branching: To demonstrate the branching process, consider the following simple Integer Program as being the root node in the B&B tree. Note that the example has been taken from [9].

Model 2 Simple Integer Program

Minimize

$$c^T x \tag{16}$$

subject to:

$$Ax = b \tag{17}$$

$$x_i \in \{a_i, a_i + 1, \ldots, b_i\} \quad \forall i \tag{18}$$

where $\{a_i, a_i + 1, \ldots, b_i\}$ is the set of all integers from a_i to b_i. The $B\&B$ procedure starts by replacing all integrality constraints (18) by $a_i \leqslant x_i \leqslant b_i$ for all i. The resulting model is the so-called *relaxed linear* program, whose solution is x_{LP}. If x_{LP} happens to satisfy integrality conditions, the algorithm is terminated because the optimal solution has been found, since integrality is satisfied without being enforced. Otherwise, a lower bound $z = c^T x_{LP}$ on the true optimal objective is obtained, since the solution to a relaxation of a minimization problem yields a lower bound. The B&B algorithm continues with the branching procedure, i.e. by picking one (usually non-integer) variable x_j , called the branching variable, and one integer $d_j \in [a_i, b_i]$. We then replace our original problem (Model 2) by two similar problems (child nodes):

Model 3 Child node 1

Minimize

$$c^T x \tag{19}$$

subject to:

$$Ax = b \tag{20}$$

$$x_i \in [a_i, b_i] \quad \forall i \neq j \tag{21}$$

$$x_j \in [a_i, d_j] \tag{22}$$

Model 4 Child node 2

Minimize

$$c^T x \tag{23}$$

subject to:

$$Ax = b \tag{24}$$

$$x_i \in [a_i, b_i] \quad \forall i \neq j \tag{25}$$

$$x_j \in [d_j + 1, b_j] \tag{26}$$

Node selection: Once the next node to solve is selected, its solution may either (i) lead to an infeasibility, in which case the node is fathomed (further exploration of that part of the tree is prohibited), or (ii) lead to an integral solution x_{int}, in which case we fathom the node and update the best objective function value found so far, $z(best)$, with $c^T x_{int}$, if it is better, or (iii) lead to a non-integral solution, x_{nint}. In the latter case, if the obtained objective value $c^T x_{nint}$ is greater than $z(best)$, then the node is dropped as it is impossible to obtain the optimal solution from that part of the solution space. If $c^T x_{nint} \leq z(best)$, then we continue branching, as we did in Model 2.

Because the solution space is finite, the algorithm will eventually terminate, either without finding an integral solution (infeasible problem), or giving us the optimal integral solution. In our case, we avoid the possibility of infeasibility; there always exist a solution which is the assignment of a separate time-slot to each link since we can use up to D time-slots (equal to the number of links in the network).

Much research in integer programming concerns how to choose the best branching and node selection strategy. However, the focus in this paper is not the methodology itself, but its application in the context of wireless networks, so we refer the reader to [10] for more details on such strategies.

V. Numerical Examples

The B&B algorithm has been coded in Microsoft Visual Studio 2005 C++ using CPLEX v11.1 and run on an Intel Core 2 computer, with 2.5GHz processor and 3.5GB of RAM.

For illustration purposes, we will use an example to demonstrate the solution obtained by solving the scheduling problem using B&B. The graph of the network is given in Figure 2 and includes $|\mathcal{T}| = |\mathcal{R}| = 10$ communication pairs. Green nodes denote the transmitters, whereas red ones denote the receivers. The colors on the links show the scheduling time for each communication pairs. Links with similar colors are active simultaneously.

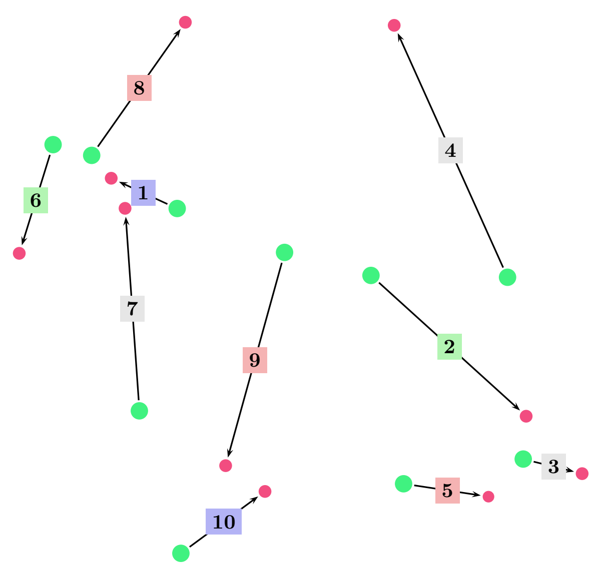


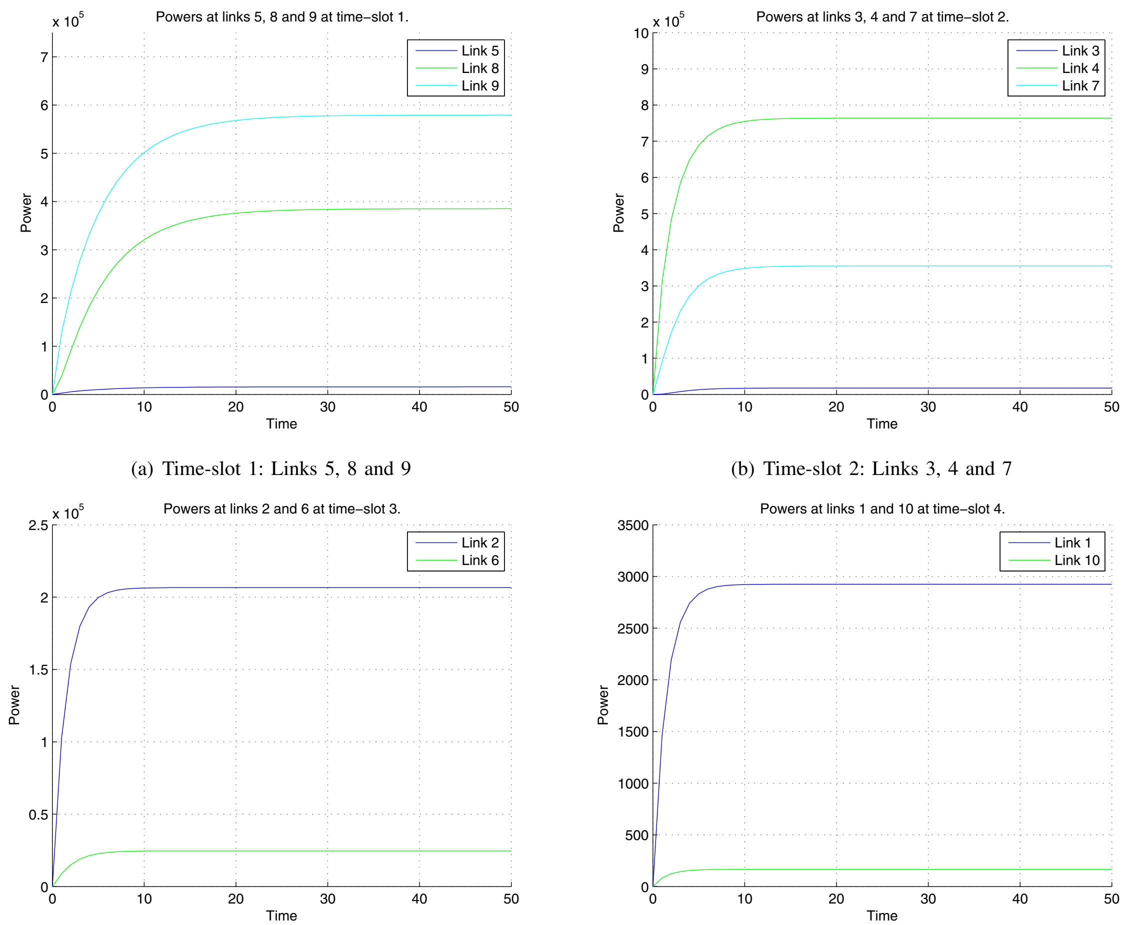
Fig. 2. Illustrative example where the transmission scheduling with SINR constraints is solved for this network.

The optimal schedule found by solving Model 1 for the above example is given in Table I and the minimum time slots required for a successful communication is 4. Note that the numbers in parentheses in Table I are the optimal power levels assigned to each communication pair.

TABLE I
Optimal Schedule for the Example Network

Time Slot	Links in process
1	5 (15820.5), 8 (385019), 9 (579226)
2	3 (17739.8), 4 (763932), 7 (355380)
3	2 (206559), 6 (24638.4)
4	1 (2924.78), 10 (166.464)

The feasibility of our solution is verified by proving that each of the subnetworks formed is a feasible network, i.e., all the communication pairs can be active simultaneously. This is done by forming matrix C, (6), and checking if the Perron-Frobenius eigenvalue is less than one.



(a) Time-slot 1: Links 5, 8 and 9

(b) Time-slot 2: Links 3, 4 and 7

(c) Time-slot 3: Links 2 and 6

(d) Time-slot 4: Links 1 and 10

Fig. 3. Distributed power control for each subnetwork verifies results.

The validity of the power levels can be found in a centralized way by

$$\mathbf{p}_{\mathbf{s}}^{\star} = (I - C_s)^{-1}\eta \tag{27}$$

where s is the time-slot considered; $\mathbf{p}_{\mathbf{s}}^{\star}$ and C_s are the vector of optimal power levels and the matrix formed by the subnetwork on time-slot s, respectively. However, once the scheduling problem is solved and the information required is sent to the communication pairs, the Foschini-Miljanic Power Control algorithm, [11], succeeds in attaining the required SINRs for all pairs in the subnetwork in a distributed manner. We demonstrate this, by simultaneously verifying the validity of our results, in Figure 3.

For our computational study we generated a set of test networks of various sizes which will serve as the benchmark for evaluating our B&B algorithm. More specifically, the benchmark set consists of networks with 10, 20, 30 and 40 pairs (we have 5 instances of each) and hence, in total, we have tested the algorithm on 20 networks. Table II shows the average computational time (in CPU seconds) required to solve the network instances as well as the percentage of instances which were solved optimally within the time limit set. We chose 3600 CPU seconds as a time limit. Note that the default settings of CPLEX were used to solve the optimization problem and no problem specific strategies to drive the solver were used.

TABLE II
NUMERICAL RESULTS OF B&B ALGORITHM FOR MODEL 1

No. of pairs	CPU time (sec)				
10	0.250	0.407	0.422	0.422	0.937
20	8.797	36.829	57.297	121.643	1655.220
30	73.126	178.204	296.160	2444.550	3600
40	75.409	900.033	3600	3600	3600

In Table II the results of our algorithm are shown. As expected, the larger the number of communication pairs considered in the network, the more difficult to solve the problems. While networks of 10 pairs are optimally solved in less than one second, networks of 20 pairs require 376 seconds on average, 30 pairs require 1318 seconds (with one not being finished) and finally, the largest class of networks, consisting of 40 pairs each, has 3 networks for which the time limit is exceeded, before the optimization problem is solved. It is conjectured that the problem is NP-hard and the proof is part of ongoing research

It is worth noting that, upon termination of the algorithm either the optimal solution is found or the time limit has been reached. In the latter case, for all the instances, the algorithm was able to provide a heuristic solution and a corresponding upper bound on the time slots required which, in most cases, differed from the lower bound by only one time slot. These results lead to the conclusion that the B&B algorithm not only is capable of solving the optimization problem under investigation but may also at least provide a heuristic schedule of good quality for the hardest instances. Improvements of the algorithm can be achieved by additional sensitivity analysis, considering various branching and node selection strategies.

VI. Conclusions

We have addressed the transmission scheduling problem in Wireless Networks with SINR constraints. In particular, using optimization techniques, we have formulated the problem of scheduling and assigning power levels to the communications pairs in a wireless network so that Quality-of-Service requirements are satisfied and the total number of time slots required is minimized.

We proposed a Branch-and-Bound solution algorithm to solve the problem under investigation efficiently. Our numerical results are based on 20 test network instances and show that the algorithm is able to solve most of the instances within the time limit set but can also provide heuristic solutions and tight lower bounds for the rest.

The significance of the results is twofold. On the one hand, we formulated and solved the problem of transmission scheduling under SINR constraints with transmitters being able to adjust their power levels to fully benefit from spatial reuse. The results are of practical importance when a central controller exists that make the calculations and disseminates the information to the rest of the network. On the other hand, the solution provides bounds for the evaluation of distributed algorithms that aim at the optimal joint power assignment and scheduling without knowledge of the whole network.

Further research is required to validate the proposed method by considering different strategies for branching and node selecting, as well as stronger lower bounds and fathoming methods. Furthermore, the next issue to be addressed is the solution of more complex problems, involving more communication pairs, in a fast and efficient manner. Finally, it would be interesting, from a practical point of view, to move from centralized into distributed algorithms, that provide optimal or near-optimal solutions.

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