# On the Stability of the Foschini-Miljanic Algorithm with Time-Delays

Themistoklis Charalambous, Ioannis Lestas and Glenn Vinnicombe
Department of Engineering,
University of Cambridge,
Cambridge CB2 1PZ, UK
(tc257, icl20, gv) @eng.cam.ac.uk

***Abstract*— Many of the distributed power control algorithms for wireless networks in the literature ignore the fact that while the algorithms necessitate communication among users, propagation delays exist in the network. This problem is of vital importance, since propagation delays are omnipresent in wireless networks. The Foschini-Miljanic algorithm is provably stable if there are no time-delays in the execution of the algorithm. However, since the interference measurements are fed back to the transmitter by its corresponding receiver, time-delays are inevitably introduced into the system. This work presents a more realistic version of the well known Foschini-Miljanic algorithm for Distributed Power Control since it considers the time-delays introduced to the system due to propagation delays. In both the continuous and discrete time cases we prove global stability of the system in the presence of propagation delays.**

## I. INTRODUCTION

Power is a valuable resource in wireless networks, since the batteries of the wireless nodes have limited lifetime. As a result, power control has been a prominent research area for all kinds of wireless communication networks (e.g. [1]–[6]). Increased power ensures longer transmission distance and higher data transfer rate. However, power minimization not only increases battery lifetime but also, the effective interference mitigation that increases the overall network capacity by allowing higher frequency reuse. Adaptive power control in wireless networks allows devices to setup and maintain wireless links with minimum power while satisfying constraints on Quality of Service (QoS).

The authors in [2] proposed a power control algorithm, the now well known as the Foschini-Miljanic (FM) algorithm, that provides for distributed on-line power control of wireless networks with user-specific Signal-to-Interference-and-Noise-Ratio (SINR) requirements. Furthermore, this algorithm yields the minimum transmitter powers that satisfy these requirements. This seminal work triggered off for numerous publications ( [3], [4] and [7], to name a few) by various authors that extended the original algorithm to account for additional issues, such as constrained power and admission control. The original algorithm along with all the extensions assume that the interference measurements at the receiver are available instantaneously at the transmitter and hence, there exists no delay while communicating these measurements. However, this is not realistic, since there are always propagation delays in the communication pairs while information is exchanged.

Our work is focused on power adaptation in an environment where there exist propagation delays between the communicating pairs. That is, we consider the fact that the interference measurement is not available at the transmitter instantaneously but with some time-delay. This consideration is important and reasonable since there are inevitable propagation delays, something that makes the model of the network more realistic. We consider the same distributed power control algorithm as in [2], but we introduce time-delays at the interference measurements of the receiver that are fed back to the transmitter. Within this setting we prove stability for the continuous-time and discrete-time FM algorithm. More specifically, using the multivariate Nyquist criterion [8] and by determining the set in which the spectrum of the multivariate system lies, we prove that both the continuous and discrete time FM algorithms are globally asymptotically stable (GAS) for arbitrarily large delays. These results indicate that the FM algorithm, compared to other power control algorithms [9], is suitable to be used in any network without requiring any bound on time-delays.

The rest of this paper is organized as follows. In the next section, the system model along with the necessary preliminary results are presented, while in section III a brief review of the FM algorithm, both continuous and discrete time, is given. In section IV the stability of those algorithms in the presence of time delays is proven. In section V, we continue with illustrative examples of the results presented. Finally, the conclusions are drawn in section VI.

## II. NOTATIONS AND PRELIMINARIES

### A. Notations

$\sigma(A)$ denotes the spectrum of matrix $A$, $\lambda(A)$ denotes an element of the spectrum of matrix $A$, and $\rho(A)$ denotes its spectral radius. $|A|$ is the elementwise absolute value of the matrix (i.e. $|A| \triangleq [|A_{ij}|]$), $A \leq B$ is the element-wise inequality between matrices A and B and $A < B$ is the strict element-wise inequality between A and B. A nonnegative matrix (i.e. a matrix whose elements are nonnegative) is denoted by $A \geq 0$ and a positive matrix is denoted by $A > 0$. $det(A)$ denotes the determinant of matrix $A$ and diag($x_i$) the matrix with elements $x_1$, $x_2$ , ... on the leading diagonal and zeros elsewhere.

### B. System Model

We consider a planar network where the links are assumed to be unidirectional and each node is supported by an omni-directional antenna. This can be represented by a graph $\mathscr{G} = (\mathscr{N}, \mathscr{L})$, where $\mathscr{N}$ is the set of all nodes and $\mathscr{L}$ is the set of the active links in the network. Each node can be a receiver or a transmitter only at each time instant due to the half-duplex nature of the wireless transceiver. Each transmitter aims to communicate with a single node (receiver) only, which cannot receive from more than one nodes simultaneously. We denote by $\mathscr{T}$ the set of transmitters and $\mathscr{R}$ the set of receivers in the network.

The channel gain on the link between transmitter $i$ and receiver $j$ is denoted by $g_{ij}$ and incorporates the mean path-loss as a function of distance, shadowing and fading, as well as cross-correlations between signature sequences. All the $g_{ij}$'s are positive and can take values in the range $(0,1]$. The power level chosen by transmitter $i$ is denoted by $p_i$ and the intended receiver is also indexed by $i$. $\nu$ denotes the variance of thermal noise at the receiver, which is assumed to be additive Gaussian noise. The link quality is measured by the Signal-to-Interference-and-Noise-Ratio (SINR). The interference power at the $i^{th}$ receiver, $I_i$, includes the interference from all the transmitters in the network (apart from the communicating transmitter) and the thermal noise, and is given by

$$I_i = \sum_{j \neq i, j \in \mathscr{T}} g_{ji} p_j + \nu. \tag{1}$$

Therefore, the SINR at the receiver $i$, $\Gamma_i$, is given by

$$\Gamma_i = \frac{g_{ii} p_i}{\sum_{j \neq i, j \in \mathscr{T}} g_{ji} p_j + \nu} \ . \tag{2}$$

Due to the unreliability of the wireless links, it is necessary to ensure Quality of Service (QoS) in terms of SINR in wireless networks. Hence, independently of nodal distribution and traffic pattern, a transmission from transmitter $i$ to its corresponding receiver is successful (error free) if the SINR of the receiver is greater or equal to $\gamma_i$ ($\Gamma_i \geq \gamma_i$), called the *capture ratio* and is dependent on the modulation and coding characteristics of the radio [10]. Therefore we require,

$$\frac{g_{ii} p_i}{\sum_{j \neq i, j \in \mathscr{T}} g_{ji} p_j + \nu} \geq \gamma_i \tag{3}$$

### C. Preliminary results

Equation (3) after manipulation, is equivalent to the following

$$p_i \geq \gamma_i \left( \sum_{j \neq i, j \in \mathscr{T}} \frac{g_{ji}}{g_{ii}} p_j + \frac{\nu}{g_{ii}} \right). \tag{4}$$

In matrix form, for a network consisting of $n$ communication pairs, this can be written as

$$\mathbf{p} \geq \Gamma G \mathbf{p} + \eta \tag{5}$$

where

$$\Gamma = diag(\gamma_i)$$
$$\mathbf{p} = \begin{pmatrix} p_1 & p_2 & \dots & p_n \end{pmatrix}^T$$
$$G_{ij} = \begin{cases} 0 & \text{, if } i = j, \\ \frac{g_{ji}}{g_{ii}} & \text{, if } i \neq j. \end{cases}$$
$$\eta_i = \frac{\gamma_i \nu}{g_{ii}}$$

Let,

$$C = \Gamma G \tag{6}$$

so that (5) can be written as

$$(I - C)\mathbf{p} \geq \eta \tag{7}$$

The matrix $C$ has nonnegative elements and it is reasonable to assume that is irreducible, since we are not considering totally isolated groups of links that do not interact with each other. By the Perron-Frobenius theorem [11], we have that the spectral radius of the matrix $C$ is a simple eigenvalue, while the corresponding eigenvector is positive component-wise. The necessary and sufficient condition for the existence of a nonnegative solution to inequality (7) for every positive vector $\eta$ is that $(I-C)^{-1}$ exists and is nonnegative. However, $(I-C)^{-1} \geq 0$ if and only if $\rho(C) < 1$ [12] (*Theorem 2.5.3*), [13] .

Therefore, the necessary and sufficient condition for (7) to have a positive solution $\mathbf{p}^*$ for a positive vector $\eta$ is that the Perron-Frobenius eigenvalue of the matrix $C$ is less than 1. That is, there exists a set of powers such that all the senders can transmit simultaneously and still meet their QoS requirements (minimum SINR for successful reception).

In this work, it is assumed that there exist separate, contention-free channels that enable the receivers to send their interference measurements to their respective transmitters.

## III. REVIEW OF THE FOSCHINI-MILJANIC ALGORITHM

The Foschini-Miljanic algorithm, [2], succeeds in attaining the required SINRs for all nodes in the network if a solution exists and fails if there does not exist a solution.

### A. The Continuous-Time Algorithm

The following differential equation is defined in [2] in order to model the continuous-time power dynamics:

$$\frac{dp_i(t)}{dt} = k_i \left( -p_i(t) + \gamma_i \left( \sum_{j \neq i, j \in \mathscr{T}} \frac{g_{ji}}{g_{ii}} p_j(t) + \frac{\nu}{g_{ii}} \right) \right) \tag{8}$$

where $k_i \in \mathbb{R}$, $k_i > 0$, denotes the proportionality constant, $g_{ji}$ denotes the channel gain on the link between transmitter $j$ and receiver $i$ and $\gamma_i$ denotes the desired SINR. It is assumed that each transmitter $i$ has knowledge of the interference at its receiver only, $I_i(t) = \sum_{j \neq i, j \in \mathscr{T}} \frac{g_{ji}}{g_{ii}} p_j(t) + \frac{\nu}{g_{ii}}$.

In matrix form this is written as

$$\dot{\mathbf{p}}(t) = -KH\mathbf{p}(t) + K\eta \tag{9}$$

where $K = diag(k_i)$ and

$$H_{ij} = \begin{cases} 1 & \text{, if } i = j, \\ -\gamma_i \frac{g_{ji}}{g_{ii}} & \text{, if } i \neq j. \end{cases}$$

For this differential equation, it is proved that the system will converge to the optimal set of solutions, $\mathbf{p}^* > 0$, for any initial power vector, $\mathbf{p}(0) > 0$. Therefore, the distributed algorithm (8) for each communication pair, leads to global stability of the system.

### B. The Discrete-Time Algorithm

As in [2], in the discrete time, we define the time coordinate so that unity is the time between consecutive power vector iterations. In correspondence with the differential equation (9), the discrete time Foschini-Miljanic algorithm is written as, [2],

$$\mathbf{p}(n+1) - \mathbf{p}(n) = -KH\mathbf{p}(n) + k\eta \quad (10)$$

The distributed power control algorithm is then given by

$$p_i(n+1) = (1-k_i)p_i(n) + k_i\gamma_i \left( \sum_{j\neq i, j\in\mathscr{T}} \frac{g_{ji}}{g_{ii}} p_j(n) + \frac{\nu}{g_{ii}} \right) \quad (11)$$

It has been shown that whenever a centralized "genie" can find a power vector, $\mathbf{p}^*$, meeting the desired criterion, then so long as the proportionality constant ($k_i$) is appropriately chosen ($k_i \in (0,1]$), then the iterative algorithm (11) converges from any initial values for the power levels of the individuals transmitters.

## IV. MAIN RESULTS

For the derivation of the main results, the following Lemma is essential:

*Lemma 1:* Let $A$ be a nonnegative square matrix ($A \in \mathbb{R}^{N\times N}$, $A \geq 0$) and $B$ be a diagonal complex matrix ($B \in \mathbb{C}^{N\times N}$) whose spectral radius $\rho(B) \leq 1$, then

$$\rho(AB) \leq \rho(A).$$

*Proof:* As background for the proof of this lemma, we need the following result [11]:

*Theorem 1:* Let $A \in \mathbb{C}^{N\times N}$ and $B \in \mathbb{R}^{N\times N}$, with $B \geq 0$. If $|A| \leq B$, then

$$\rho(A) \leq \rho(|A|) \leq \rho(B).$$

Note that $AB \leq |AB| \leq |A||B|$. Since $A$ is a nonnegative matrix with real entries, then $|A| = A$. In addition $|B| \leq I$, where $I$ is the identity matrix of appropriate dimensions. Therefore, $|A||B| \leq A$. Thus, from Theorem 1,

$$\rho(AB) \leq \rho(|AB|) \leq \rho(|A||B|) \leq \rho(A).$$

■

### A. The Continuous-Time Foschini-Miljanic Algorithm with Time-Delays

Since the transmitter uses information (interference) provided by the receiver, unavoidably, there exists a time-delay on the information used while updating the power. Consequently, for a more realistic algorithm we introduce delays to the continuous-time FM algorithm and analyze the stability conditions for this system. The differential equation (8), when the time-delay is introduced becomes

$$\frac{dp_i(t)}{dt} = k_i \left( -p_i(t) + \gamma_i \left( \sum_{j\neq i, j\in\mathscr{T}} \frac{g_{ji}}{g_{ii}} p_j(t-T_i) + \frac{\nu}{g_{ii}} \right) \right). \quad (12)$$

The following theorem states that if the system is stable when there are no delays into the network, then it is also stable for arbitrarily large time-delays, $T_i > 0$, and for any proportionality constant, $k_i > 0$. Note that, at $p_i(t) = 0$, from (12) $dp_i(t)/dt > 0$ restricting the power to be nonnegative, thus fulfilling the physical constraint that the power $p_i \geq 0$. Hence, we should not worry about saturation issues in the system.

*Theorem 2:* If the spectral radius of matrix $C$ in (6) is less than 1, then the following power control algorithm

$$\frac{dp_i(t)}{dt} = k_i \left( -p_i(t) + \gamma_i \left( \sum_{j\neq i, j\in\mathscr{T}} \frac{g_{ji}}{g_{ii}} p_j(t-T_i) + \frac{\nu}{g_{ii}} \right) \right), \ i \in \mathscr{T}$$

for $\gamma_i$, $g_{ji}$, $\nu > 0$, is asymptotically stable for arbitrarily large delays, $T_i > 0$, for any initial state $p_i(0) > 0$ and for any proportionality constant, $k_i > 0$.

*Proof:* Taking Laplace transforms of differential equation (12),

$$sP_i(s) - p_i(0) = -k_i \left[ P_i(s) - \gamma_i \left( \sum_{j\neq i, j\in\mathscr{T}} \frac{g_{ji}}{g_{ii}} e^{-sT_i} P_j(s) + \frac{\nu}{sg_{ii}} \right) \right]$$

$$(s+k_i)P_i(s) - k_i \left[ \gamma_i \left( \sum_{j\neq i, j\in\mathscr{T}} \frac{g_{ji}}{g_{ii}} e^{-sT_i} P_j(s) \right) \right] = p_i(0) + \frac{k_i\nu}{sg_{ii}}$$

which can be written as

$$P_i(s) - \frac{k_i e^{-sT_i}}{s+k_i} \left[ \gamma_i \left( \sum_{j\neq i, j\in\mathscr{T}} \frac{g_{ji}}{g_{ii}} P_j(s) \right) \right] = \frac{sg_{ii}p_i(0) + k_i\nu}{sg_{ii}(s+k_i)}$$

In matrix form this is written as

$$(I - T(s)C)\mathbf{P}(s) = \mathbf{f}(s) \quad (13)$$

where

$$T(s) = diag(\frac{k_i e^{-sT_i}}{s+k_i})$$

$$f_i(s) = \frac{sg_{ii}p_i(0) + k_i\nu}{sg_{ii}(s+k_i)}$$

The closed-loop system is stable if $\det(I - T(s)C)$ has no zero in the closed right-half plane. From the multivariate Nyquist criterion, [8], it is sufficient to show that the eigenvalues of $T(j\omega)C$ do not encircle the $+1$ point. Note that

$$\left| \frac{k_i}{j\omega + k_i} \right|_{\omega\in\mathbb{R}_+} \leq 1, \ \forall \ k_i > 0$$

and $|e^{-j\omega T_i}|_{\omega \in \mathbb{R}_+} = 1 \ \forall \ T_i > 0$. Therefore,

$$\left\{ \frac{k_i}{j\omega + k_i} e^{-j\omega T_i} : \omega, k_i, T_i \in \mathbb{R}_+ \right\} \subseteq S$$

where $S$ is the unit ball, i.e. $S = \{x : \| x \| \leq 1\}$. Since,

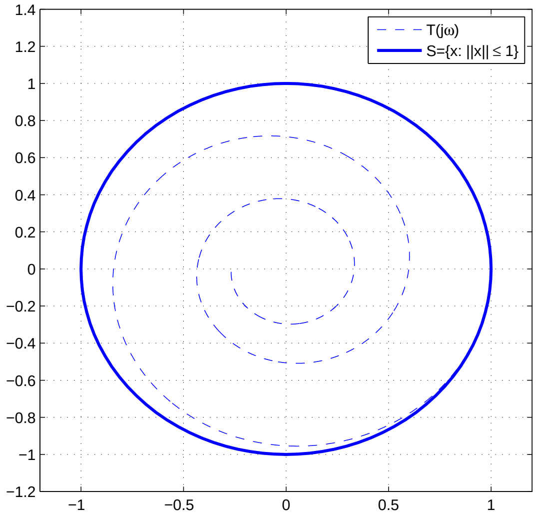


Fig. 1. The union of $T(j\omega)$ is bounded by the unit ball. This is illustrated by an example in this figure for $k_i = 2$ and $T_i = 2$ $\left( \left\{ \frac{2}{j\omega+2} e^{-2j\omega} : \omega, \in \mathbb{R}_+ \right\} \subseteq S \right)$.

$\sigma(CT(j\omega)) = \sigma(T(j\omega)C)$, then it is sufficient to prove that the spectral radius of $CT(j\omega)$ is smaller than 1. From Lemma 1, we establish that

$$\rho(CT(j\omega)) \leq \rho(C).$$

Using the fact that $\rho(C) < 1$, then $\rho(CT(j\omega)) < 1$. Therefore, the spectrum is bounded within the unit ball and does not encircle the point $+1$. Therefore the closed-loop system is stable for arbitrarily large delays and for any proportionality constant $(k_i)$. ∎

*Remark 1:* Theorem 2 proves that (12) is globally asymptotically stable if and only if the Perron-Frobenius eigenvalue of matrix C is less than 1. However, a sufficient condition to establish stability to the system without requiring the knowledge of the whole matrix $C$, could be $\| C \|_\infty < 1$, i.e.

$$\frac{g_{ii}}{\sum_{j \neq i, j \in \mathcal{T}} g_{ji}} > \gamma_i \qquad \forall \ i. \tag{14}$$

Since $\rho(C) \leq \| C \|_\infty$, this condition is more conservative. This condition is equivalent to $H$ being a diagonally dominant matrix with all main diagonal entries being positive. Hence, this guarantees that all the eigenvalues of matrix $H$ have positive real part, [11]. It, therefore, provides an upper bound on the achievable target SINR levels in a given network, and hence, leads to a soft capacity constraint for the underlying system. The return for this conservatism is that the only extra information required at each transmitter is a measure of the sum of the channel gains at its receiver by all other transmitters. Hence, we are able to use a distributed way of updating the desired SINR levels and keep the network functioning. In case a communication pair cannot reach its desired SINR and cannot be compromised by a lower SINR level, then the transmitter may wish to either back-off until condition (14) is satisfied for a reasonable SINR level, or go closer to the receiver, if possible (i.e. increase $g_{ii}$).

### *B. The Discrete-Time Foschini-Miljanic Algorithm with Time-Delays*

With similar arguments, we now study the behavior of the Discrete Time Foschini-Miljanic Algorithm when we introduce delays. The difference equation now becomes

$$p_i(n+1) = (1-k_i)p_i(n) + k_i\gamma_i \left( \sum_{j \neq i, j \in \mathcal{T}} \frac{g_{ji}}{g_{ii}} p_j(n-n_i) + \frac{\nu}{g_{ii}} \right) \tag{15}$$

where $n_i \in \mathbb{N}$ denotes the time delays.

We prove that the stability is maintained whatever the delay introduced into the network, provided that the proportionality constant lies within the interval $k_i \in (0,1]$. Note that, since $k_i \leq 1$, from (15) it is obvious that $p_i(n+1)$ is always nonnegative. Thus the physical constraint that the power $p_i \geq 0$ is fulfilled.

*Theorem 3:* If the spectral radius of matrix $C$ in (6) is less than 1, then the discrete time algorithm

$$p_i(n+1) = (1-k_i)p_i(n) + k_i\gamma_i \left( \sum_{j \neq i, j \in \mathcal{T}} \frac{g_{ji}}{g_{ii}} p_j(n-n_i) + \frac{\nu}{g_{ii}} \right), \ i \in \mathcal{T}$$

for $\gamma_i$, $g_{ji}$, $\nu > 0$, is asymptotically stable for arbitrarily large delays ($n_i \in \mathbb{N}$) to the system, for any initial state $p_i(0) > 0$ and for $k_i$ appropriately chosen ($k_i \in (0,1]$).

*Proof:* Taking z-Transforms to the discrete time algorithm (15), we have

$$zP_i(z) - p_i(0) = (1-k_i)P_i(z) + k_i\gamma_i \left( \sum_{j \neq i, j \in \mathcal{T}} \frac{g_{ji}}{g_{ii}} P_j(z) z^{-n_i} + \frac{\nu z}{g_{ii}(z-1)} \right)$$

$$(z-1)\left[ (z-1+k_i)P_i(z) - k_i\gamma_i \left( \sum_{j \neq i, j \in \mathcal{T}} \frac{g_{ji}}{g_{ii}} P_j(z) z^{-n_i} \right) \right] = k\gamma_i \frac{z\nu}{g_{ii}} + (z-1)p_i(0)$$

In matrix form this is written as

$$(z-1)F(z)P(z) = f(z) \tag{16}$$

where

$$f_i(z) = k\gamma_i \frac{z\nu}{g_{ii}} + (z-1)p_i(0),$$

$$F_{ij}(z) = \begin{cases} z-1+k_i & , \text{if } i = j, \\ -k_i\gamma_i \frac{g_{ji}}{g_{ii}} z^{-n_i} & , \text{if } i \neq j. \end{cases}$$

Note that $z = 1$ cannot be a solution to

$$(z-1)F(z)P(z) = f(z). \tag{17}$$

$F(z)$ can be written as $F(z) = (z-1)I + K - KD(z)C$ where $D(z) = \mathrm{diag}(z^{-n_i})$. So, the stability condition is equivalent to the following: the eigenvalues of

$$[(e^{j\theta}-1)I + K]^{-1} KD(e^{j\theta})C \tag{18}$$

should not encircle the point $+1$, as $\theta$ varies from 0 to $2\pi$. In order to establish that the spectral radius of

$$\tilde{D}(e^{j\theta}) = [(e^{j\theta}-1)I+K]^{-1}KD(e^{j\theta}) \qquad (19)$$

is upper-bounded by 1, it is sufficient to find the conditions for $k_i$ for which

$$\left|\frac{k_i}{e^{j\theta}-1+k_i}\right| \leq 1 \; \forall \; i \in \mathscr{T}. \qquad (20)$$

Since $k_i > 0$, inequality (20) can be written as

$$\begin{aligned} k_i &\leq |e^{j\theta}-1+k_i| \\ &= |(\cos\theta + k_i - 1) + j\sin\theta| \\ &= \sqrt{(\cos\theta+k_i-1)^2+\sin^2\theta} \end{aligned}$$

Thus,

$$\begin{aligned} k_i^2 &\leq (\cos\theta+k_i-1)^2+\sin^2\theta \\ &= \cos^2\theta + 2(k_i-1)\cos\theta + (k_i-1)^2 + \sin^2\theta \\ &= (k_i-1)^2 + 2(k_i-1)\cos\theta + 1 \end{aligned}$$

Hence,

$$(k_i-1)(2-2\cos\theta) \leq 0$$

But, since $|\cos\theta| \leq 1 \;\; \forall\theta \in [0,2\pi]$, then $2-2\cos\theta \geq 0$. Therefore, this inequality holds for $k_i \leq 1$. Again, since $\sigma(C\tilde{D}(e^{j\theta})) = \sigma(\tilde{D}(e^{j\theta})C)$, then it is sufficient to prove that the spectral radius of $C\tilde{D}(e^{j\theta})$ is smaller than 1 (i.e. $\rho(C\tilde{D}(e^{j\theta})) < 1$). From Lemma 1, we establish that

$$\rho(C\tilde{D}(e^{j\theta})) \leq \rho(C).$$

Since $\rho(C) < 1$, then $\rho(C\tilde{D}(e^{j\theta})) < 1$. Therefore, the spectrum is bounded within the unit ball and does not encircle the point +1. Therefore the closed-loop system is stable for arbitrarily large delays and for $k_i \leq 1$. ■

*Remark 2:* From (18), since $|D(e^{j\theta})| = I$, the stability condition is equivalent to establishing that the spectral radius of

$$\tilde{C}(e^{j\theta}) = C[(e^{j\theta}-1)I+K]^{-1}K \qquad (21)$$

is smaller than 1. Since $[(e^{j\theta}-1)I+K]^{-1}K$ is a diagonal matrix, by Lemma 1, we can equivalently find the conditions on $k_i$ for which

$$\left|\frac{k_i\rho(C)}{e^{j\theta}-1+k_i}\right| < 1 \; \forall \; i \in \mathscr{T}. \qquad (22)$$

Since $k_i > 0$ and $\rho = \rho(C) > 0$, inequality (22) can be written as $k_i\rho < |e^{j\theta}-1+k_i|$. Thus,

$$k_i^2\rho^2 < (k_i-1)^2 + 2(k_i-1)\cos\theta + 1, \qquad (23)$$

which is equivalent to the following inequality

$$k_i^2(\rho^2-1) + 2k_i - 2 < 2(k_i-1)\cos\theta. \qquad (24)$$

For $k_i \leq 1$, the worst case is obtained when $\cos\theta = 1$. Therefore, from inequality (24), we get $\rho < 1$. When $k_i \leq 1$ then, if the system is stable, the algorithm will converge in a distributed manner.

For $k_i > 1$, the worst case is obtained when $\cos\theta = -1$. Therefore, from inequality (24), we obtain that

$$k_i \in \left(0, \frac{2}{1+\rho(C)}\right) \bigcup \left(\frac{2}{1-\rho(C)}, +\infty\right)$$

However, for values of $k_i > 2$ the system (17) becomes open-loop unstable. Therefore for

$$k_i \in \left(0, \frac{2}{1+\rho(C)}\right),$$

if the system is stable (i.e. $\rho(C) < 1$) then the algorithm is *locally* asymptotically stable. Global stability of the system requires to prove stability for the algorithm while restricting negative powers, which is part of an ongoing research.

This result is useful whenever there is a centralized controller/base station that has knowledge of the network and is able to disseminate this information to the users. In such a case the system converges faster to the optimal power vector.

## V. ILLUSTRATIVE EXAMPLE

Consider an ad-hoc network consisting of four communicating pairs, i.e. eight mobile devices in total. For this example we have that $\gamma_i = 3$ and $\nu = 0.04$ *Watts*. The initial power $p_i(0)$ for each transmitter is 1 *Watt*. The network is described by matrix $C$ and it is schematically shown in Figure 2.

$$C = \begin{pmatrix} 0 & 0.5405 & 0.3880 & 0.1131 \\ 0.2143 & 0 & 0.0101 & 0.0323 \\ 0.0522 & 0.0070 & 0 & 0.0271 \\ 0.0084 & 0.0016 & 0.0385 & 0 \end{pmatrix}$$

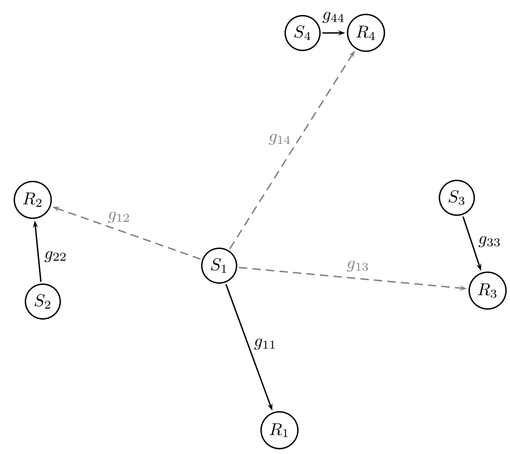


Fig. 2. Example of a wireless ad-hoc network of $n = 8$ nodes, consisting of four communication pairs $\{S_i \to R_i\}$. The grey dotted arrows are included to indicatively show the interference caused to the receivers by $S_1$.

For this setup, the Perron-Frobenius eigenvalue of $C$ is 0.3759, so the power control algorithm is stable, even though $\| C \|_\infty > 1$.

For the same network, utilizing the discrete-time FM algorithm, the system is asymptotically stable, provided the proportionality constant is appropriately chosen such that

$k_i \in (0,1]$. This is demonstrated in the simulation below, Figure 3, for a proportionality constant $k_i = 1$ and different time-delays for each communication pair. In a distributed

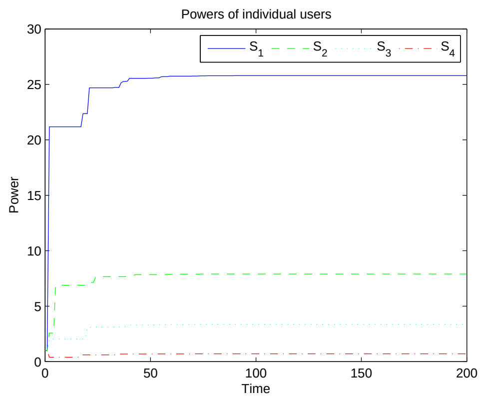


Fig. 3. Discrete-time FM algorithm with delays ($T = \{15, 2, 17, 14\}$). The algorithm asymptotically converges to the desired SINR in a distributed manner.

implementation of the algorithm where all transmitters satisfy (14), assuming that the nodes acquire the information required for updating their desired SINRs, the first communicating pair has to reduce the data rate, and hence require smaller SINR, such that

$$\sum_j C(i,j) < 1, \text{i.e., } \gamma_1 < 2.8802.$$

When there are no delays into the system, then for the network described by matrix $C$, the maximum proportionality constant for which the system is locally asymptotically stable is given by

$$k < \frac{2}{1+\rho(C)} = \frac{2}{1.3659} = 1.4643.$$

In order to indicate the validity of the result, we run simulations for the network in which all users have a proportionality constant $k = 1.4$ (Figure 4) and then $k = 1.5$ (Figure 5).

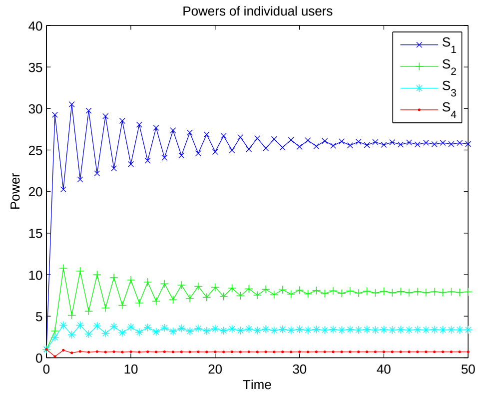


Fig. 4. Proportionality constant $k = 1.4$. The system converges to the desired SINR and to the minimum power vector.

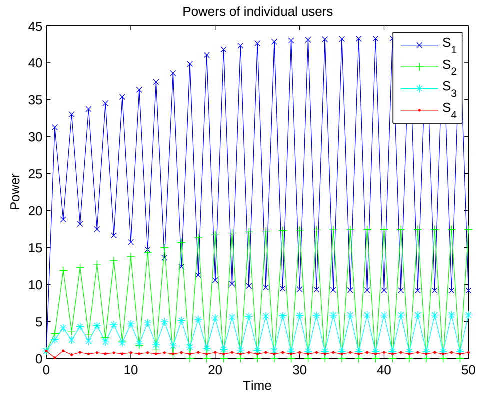


Fig. 5. Proportionality constant $k = 1.5$. The system diverges, but not to infinity since the lower bound of the powers is restricted to be positive.

## VI. Conclusions

In this paper, we focused on power adaptation in an environment where there exist propagation delays between the communicating pairs. More specifically, we introduced delays to both the continuous-time and discrete-time Foschini-Miljanic algorithm. Using the multivariate Nyquist criterion we proved global asymptotic stability for both algorithms in the presence of arbitrarily large delays. In the continuous-time case, the proportionality constants should be positive, while in the discrete-time case, the proportionality constants should lie in the half-open interval $(0, 1]$.

## References


[1] J. Zander, "Distributed co-channel interference control in cellular radio systems," *IEEE Transaction on Vehicular Technology*, vol. 41, no. 3, pp. 305–311, August 1992.

[2] G. Foschini and Z. Miljanic, "A Simple Distributed Autonomous Power Control Algorithm and its Convergence," *IEEE Transactions on Vehicular Technology*, vol. 42, no. 4, pp. 641–646, November 1993.

[3] S. Grandhi, J. Zander, and R. Yates, "Constrained power control," *Wireless Personal Communications*, vol. 2, no. 3, August 1995.

[4] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal on Selected Areas in Communications*, vol. 13, pp. 1341–1347, September 1995.

[5] T. ElBatt and A. Ephremides, "Joint Scheduling and Power Control for Wireless Ad-hoc Networks," in *Proceedings of IEEE INFOCOM*, 2002.

[6] Z. Gajic, D. Skataric, and S. Koskie, "Optimal SIR-based Power Updates in Wireless CDMA Communication Systems," in *IEEE Conference on Decision and Control*, vol. 5, December 2004, pp. 5146–5151.

[7] N. Bambos, S. C. Chen, and G. J. Pottie, "Channel Access Algorithms with Active Link Protection for Wireless Communication Networks with Power Control," *IEEE/ACM Transactions on Networking*, vol. 8, no. 5, pp. 583–597, 2000.

[8] C. A. Desoer and Y. Yang, "On the generalized Nyquist stability criterion," *IEEE Transaction on Automatic Control*, vol. 25, no. 1, pp. 187–196, 1980.

[9] X. Fan, M. Arcak, and J. Wen, "Robustness of CDMA power control against disturbances and time-delays," in *American Control Conference*, vol. 3, 2004, pp. 3622–3627.

[10] B. Krishnamachari, *Networking Wireless Sensors*. New York, USA: Cambridge University Press, 2005.

[11] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.

[12] ——, *Topics in Matrix Analysis*. Cambridge University Press, 1994.

[13] S. Pillai, T. Suel, and S. Cha, "The Perron-Frobenius theorem: some of its applications," *Signal Processing Magazine, IEEE*, vol. 22, pp. 62–75, March 2005.